MIDTERM EXAM #1
Physics 1A
Instructor: Anton Bondarenko

Friday, February 9th, 2018
9:00 AM – 9:50 AM

Last name: ______________________________

First name: ______________________________

University ID: ____________________________

You will have **50 minutes** to complete this exam. One standard 3" x 5" index card and a calculator are permitted. **Notes, books, cell phones, and any other electronics are not allowed.**

Please write your answer in the space below the problem. **You must write legibly and demonstrate your reasoning to get full credit.** For quantitative problems, always express your final answer in terms of the variables given in the problem unless otherwise stated. For clarity, please draw a box around your final answer.

Extra paper is attached at the back of the exam and more is available in the front of the room. If a problem seems confusing or ambiguous, please ask the proctor for clarifications.

PLEASE DO NOT TURN PAGE UNTIL INSTRUCTED

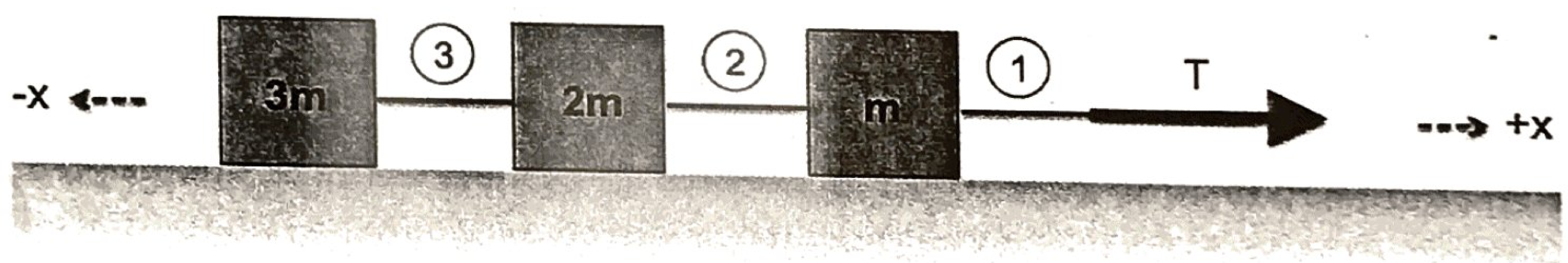
Problem 1 (40 points):

As shown in **Figure 1** below, three blocks of mass m, $2m$, and $3m$ slide along a frictionless horizontal surface. The blocks are connected by identical strings of negligible mass, labeled 1, 2, and 3. The block-string system is sliding in the $-x$ direction at constant speed when, at time $t = 0$, a time-dependent tension force of magnitude T is applied in the $+x$ direction to string 1. For times $t \geq 0$, the velocity vector of any point on the block-string system is found to be described by the function

$$\vec{v}(t) = (\alpha t^2 - \beta)\hat{i}\ ,$$

where α and β are positive constants. For this problem, you can assume that every part of the block-string system moves together and that the strings connecting the blocks are stretched out to their maximum lengths for all times $t \geq 0$.

Figure 1



Part A (10 points): Determine the acceleration vector of any point on the block-string system as a function of time.

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t) = \boxed{2\alpha t\,\hat{i}}$$

Part B (10 points): A specific point of the block-string system is found to pass the origin at time $t = 0$. Determine the position vector of this point as a function of time.

$$\vec{r}(t) = \int_0^t \vec{v}(\tau)\,d\tau + 0$$

$$= \int_0^t (\alpha\tau^2 - \beta)\hat{i}\,d\tau$$

$$= \boxed{\left(\frac{1}{3}\alpha t^3 - \beta t\right)\hat{i}}$$

Part C (10 points): What is the tension magnitude in string 1 at the time when the block-string system is instantaneously at rest? Express your answer in terms of m, α, and β.

At rest: $\vec{v}(t) = \vec{0}$

$$\alpha t^2 - \beta = 0$$

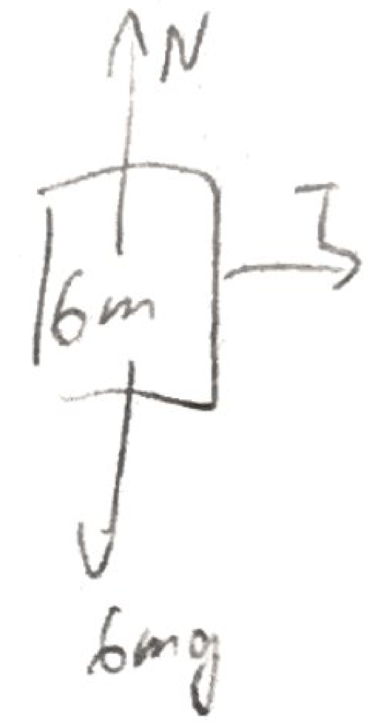
$$\alpha t^2 = \beta$$

$$t^2 = \frac{\beta}{\alpha}$$

$t > 0$

$$t = \sqrt{\frac{\beta}{\alpha}}$$

$$\vec{a}\left(\sqrt{\tfrac{\beta}{\alpha}}\right) = 2\alpha\sqrt{\beta/\alpha}\ \hat{i}$$



$$\Sigma\vec{F} = \vec{T}_1 = (6m)\vec{a}$$

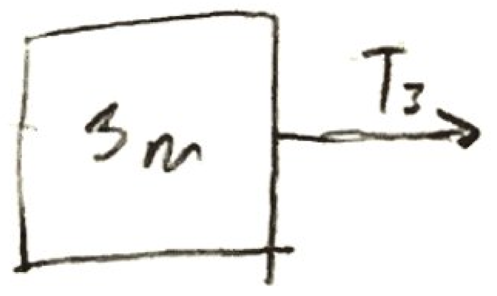
$$= 12m\alpha\sqrt{\beta/\alpha}\ \hat{i}$$

$$= 12m\sqrt{\alpha\beta}\ \hat{i}.$$

$$|\vec{T}_1| = \boxed{12m\sqrt{\alpha\beta}}$$

Part D (10 points): What is the tension magnitude in string 3 at the time when the block-string system is instantaneously at rest? Express your answer in terms of *m, α,* and β.

$$\vec{a} = 2\alpha\sqrt{\frac{\beta}{\alpha}}\,\hat{i} = 2\sqrt{\alpha\beta}\,\hat{i}$$



$$\sum \vec{F} = \vec{T}_3 = (3m)\,\vec{a}$$

$$= 6m\sqrt{\alpha\beta}\,\hat{i}$$

$$|\vec{T}_3| = \boxed{6m\sqrt{\alpha\beta}}$$

Problem 2 (40 points):

A daredevil wants to design a catapult that will launch him from one incline to another, as shown in **Figure 2** below. Both inclines are identical, having a base length L and an angle of $\alpha_0 = 45°$. The horizontal distance between the inclines is $2L$. The daredevil initially stands on a platform at the bottom of the incline on the left. The combined mass of the daredevil and platform is M_1. The platform is connected via a rope and pulley to a vertically hanging counterweight of mass M_2. When the counterweight is released from rest, it descends and pulls the platform and daredevil up the incline. At the very top of the incline, the platform is stopped suddenly, launching the daredevil. The acceleration magnitude due to gravity is g.

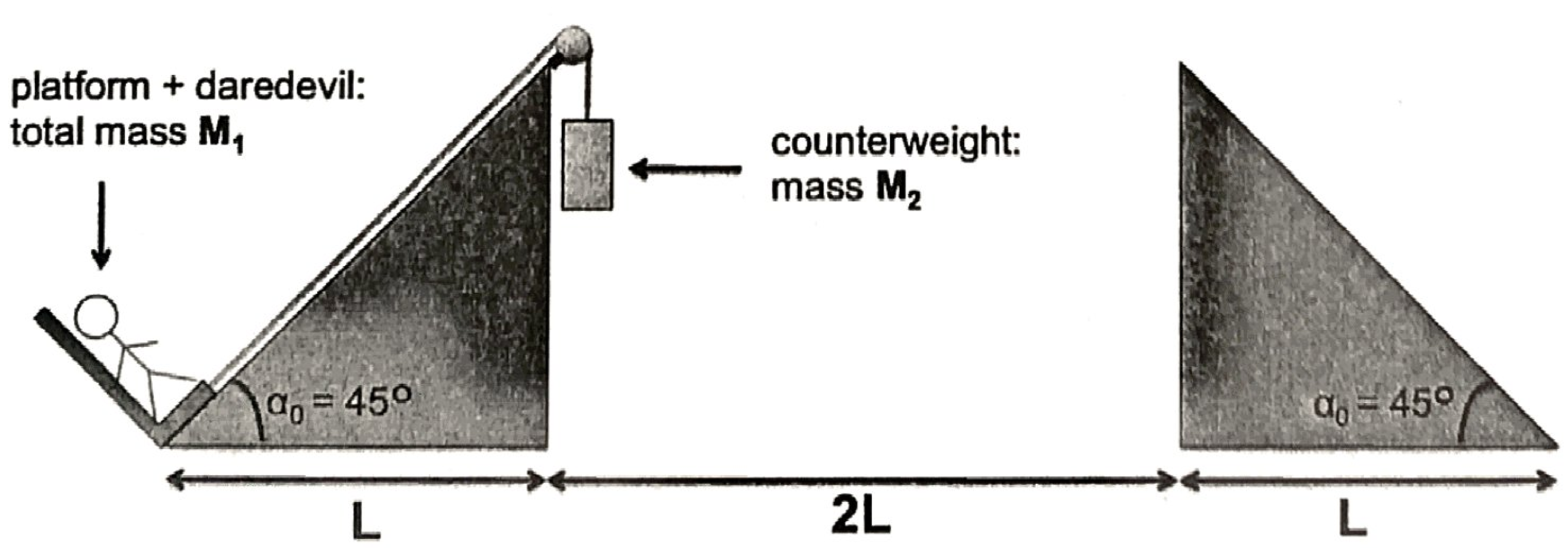
For this problem, you can assume the following:

- The daredevil is launched from the very top of the incline at the same speed that the platform attains just before it is stopped suddenly.
- The size of the platform is negligible with respect to the size of the incline, such that the distance that the platform travels can be approximated as the entire length of the incline and the height from which the daredevil is launched can be approximated as the height of the incline.
- The friction between the platform and incline is negligible.
- The mass of the rope and pulley and the friction of the pulley are negligible.
- Air resistance is negligible.
- The counterweight doesn't hit the ground before the daredevil is launched (e.g., it can keep descending into a hole in the ground).

To simplify your answers as much as possible, make use of:

$$\sin 90° = 1 \quad \sin 45° = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

Figure 2



10

Part A (10 points): What is the minimum speed with which the daredevil must be launched from the top of the incline on the left side in order to reach the top of the incline on the right side? Express your answer in terms of L and g.

x-direction: $v_0 \cos 45^\circ \, t = 2L.$

$$v_0 \cdot \frac{\sqrt{2}}{2} \, t = 2L$$

$$t = 2L \cdot \sqrt{2} \cdot \frac{1}{v_0}$$

y-direction: $v_0 \sin 45^\circ \, t - \frac{1}{2} g t^2 = 0$

$$v_0 \cdot \frac{\sqrt{2}}{2} \cdot \left(2L \sqrt{2} \cdot \frac{1}{v_0}\right) = \frac{1}{2} g \left(4L^2 \cdot 2 \cdot \frac{1}{v_0^2}\right)$$

$$2L = 4gL^2 \cdot \frac{1}{v_0^2}$$

$$v_0 = \sqrt{\frac{4gL^2}{2L}} = \boxed{\sqrt{2gL}}$$

$$v_0 \frac{\sqrt{2}}{2} \cdot t = \frac{1}{2} g t$$

$$\frac{v_0}{\sqrt{2}} = \frac{1}{2} g \cdot 2L \cdot \sqrt{2} \cdot \frac{1}{v_0}$$

$$v_0^2 = 2gL$$

$$v_0 = \sqrt{2gL}$$

Part B (10 points): What acceleration magnitude must the platform have as it ascends the incline in order to attain the speed of Part A? Express your answer in terms of g.

10

$$v^2 = v_0^2 + 2a\,\Delta x$$

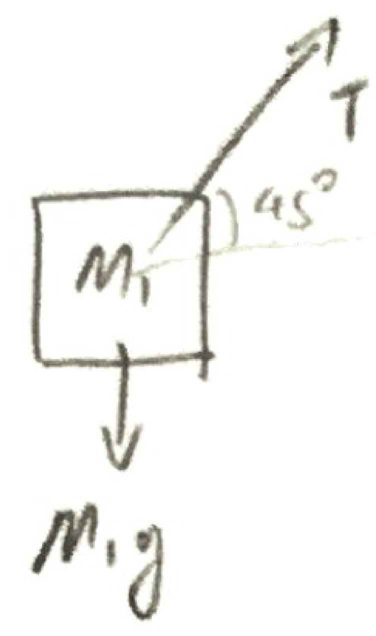
$$\cancel{2}gk = \cancel{2}a \cdot \frac{k}{\sin 45^\circ}$$

$$a = g \sin 45^\circ$$

$$a = \boxed{\frac{2g}{\sqrt{2}}}$$

4

Part C (10 points): What tension magnitude is required in the rope in order to achieve the acceleration magnitude from Part B? Express your answer in terms of M_1 and g.



$$a_x = \frac{T\cos 45^\circ}{m_1} = \frac{T}{\sqrt{2}\, m_1}$$

$$a_y = \frac{T\sin 45^\circ - m_1 g}{m_1} = \frac{T}{m_1\sqrt{2}} - g.$$

$$a = \sqrt{\frac{T^2}{2m_1^2} + \frac{T^2}{2m_1^2} - \frac{2gT}{m_1\sqrt{2}} + g^2}$$

$$= \sqrt{\frac{T^2}{m_1^2} - \frac{\sqrt{2}\,gT}{m_1} + g^2} = \frac{2g}{\sqrt{2}}$$

$$\frac{T^2}{m_1^2} - \frac{\sqrt{2}\,gT}{m_1} + g^2 = \frac{g^2}{2}$$

$$T^2 - m_1\sqrt{2}\,gT + \frac{m_1^2 g^2}{2} = 0.$$

$$T = \frac{m_1\sqrt{2}\,g \pm \sqrt{m_1^2 \cdot 2 \cdot g^2 - 4/2\, m_1^2 g^2}}{2}$$

$$= \frac{m_1\sqrt{2}\,g + \cancel{\sqrt{2m_1^2 g^2 - 2m_1^2 g^2}}}{2}$$

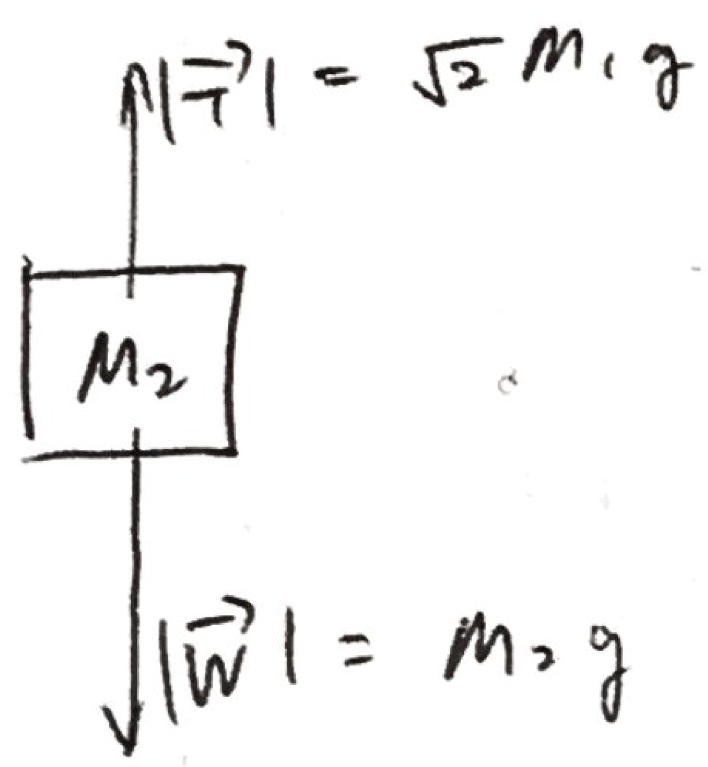
$$= \boxed{\frac{m_1\sqrt{2}\,g}{2}}$$

2

Part D (10 points): What counterweight mass M_2 is required in order to achieve the acceleration magnitude from Part B? Express your answer in terms of M_1.

$$T = \frac{M_1 \sqrt{2} \cancel{g}}{2} = M_2 g$$

$$M_2 = \boxed{\frac{M_1 \sqrt{2}}{2}}$$



$$|\vec{W}| - |\vec{T}| = (M_2 - \sqrt{2} M_1) g = M_2 \frac{g}{\sqrt{2}}$$

$$\cancel{M_2 - \frac{1}{\sqrt{2}} M_2 = \sqrt{2} M_1}$$

$$\cancel{\frac{1}{\sqrt{2}} M_2 - \frac{1}{2} M_2 = M_1}$$

$$\cancel{\boxed{\frac{\sqrt{2} - 1}{2} M_2 = M_1}}$$

$$M_2 = \frac{\sqrt{2} M_1}{1 - \frac{1}{\sqrt{2}}}$$

$$= \boxed{\frac{2 M_1}{\sqrt{2} - 1}}$$

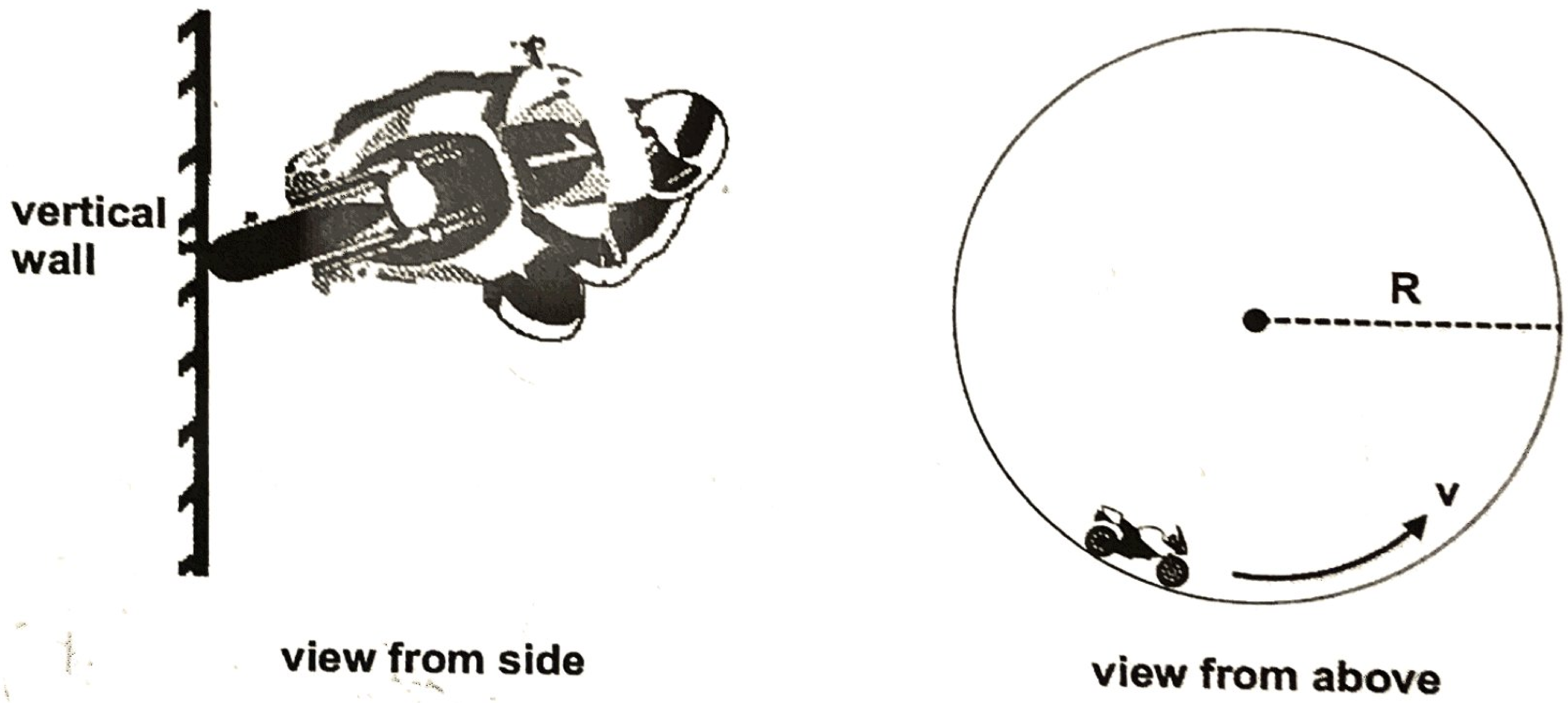
$$= \frac{2 M_1}{\sqrt{2} - 1} \cdot \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$

$$= \boxed{2 (\sqrt{2} + 1) M_1}$$

Problem 3 (20 points):

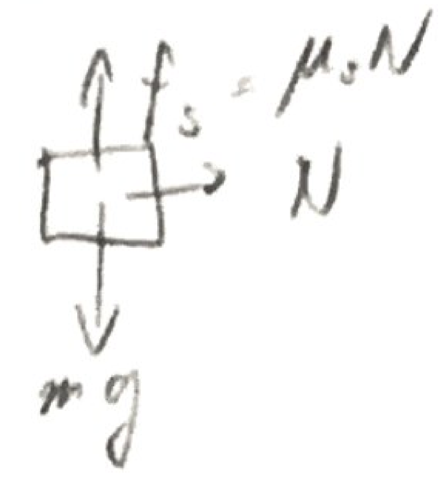
As shown in **Figure 3**, a motorcyclist rides on the inside of a vertical cylindrical wall of radius R at constant speed v. The coefficient of static friction between the motorcycle's tires and the wall is μ_s. For this problem, the sizes of the motorcyclist and motorcycle can be assumed negligible with respect to the radius of the cylindrical wall.

Figure 3



Part A (10 points): What is the longest amount of time that the motorcyclist can take in order to complete one revolution without slipping off of the wall? Express your answer in terms of R, μ_s, g and fundamental constants.

$$a = \frac{v^2}{R}$$



$$N = ma = \frac{mv^2}{R}$$

$$\mu_s N = mg$$

$$\mu_s \cdot \frac{mv^2}{R} = mg$$

$$v^2 = \frac{gR}{\mu_s}$$

$$v = \sqrt{\frac{gR}{\mu_s}}$$

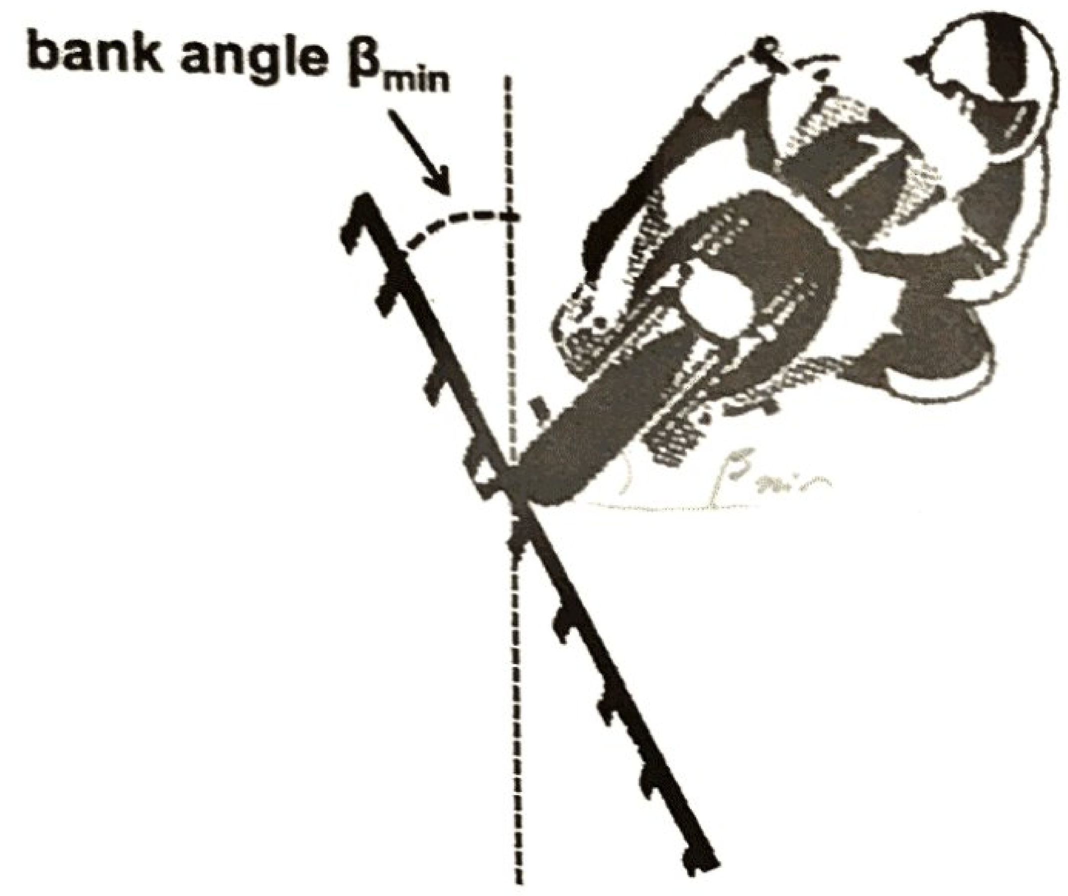
diameter $= 2\pi R$.

$$T = \frac{2\pi R}{\sqrt{\frac{gR}{\mu_s}}} = \frac{2\pi R\sqrt{\mu_s}}{\sqrt{gR}} = \frac{2\pi\sqrt{R\mu_s}}{\sqrt{g}} = \boxed{2\pi\sqrt{\frac{R\mu_s}{g}}}$$

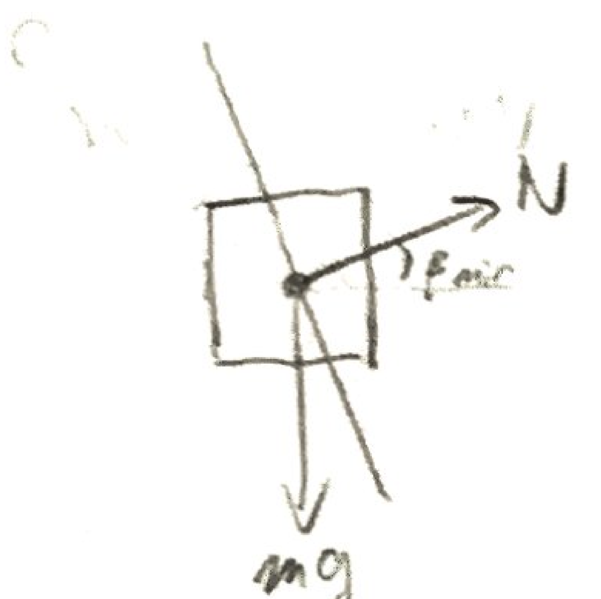
10

Part B (10 points): Assume that the wall suddenly becomes extremely icy, effectively lowering the coefficient of static friction μ_s to zero. What minimum bank angle β_{min} with respect to the vertical (see **Figure 4** below) would the wall now need in order for the motorcyclist to maintain a constant speed v at a radius R without slipping off the wall? Express your answer in terms of R, g, and v.

Figure 4



view from side



$$N \sin \beta_{min} = mg.$$

$$N \cos \beta_{min} = \frac{v^2}{R} m$$

$$\tan \beta_{min} = \frac{gR}{v^2} \qquad \beta_{min} < 90°$$

$$\beta_{min} = \boxed{\tan^{-1} \frac{gR}{v^2}}.$$

(extra page)

10

SCORING

Problem 1:

Part A: 10 / 10
Part B: 10 / 10
Part C: 10 / 10
Part D: 10 / 10
Total: 40 / 40

Problem 2:

Part A: 10 / 10
Part B: 10 / 10
Part C: 4 / 10
Part D: 2 / 10
Total: 26 / 40

Problem 3:

Part A: ______ / 10
Part B: ______ / 10
Total: 20 / 20

Total Midterm #1 Score 86 / 100