

Problem 1

A small block, initially at rest at point A, slides down a curve leading to a half-circle of radius R without friction. (Express all answers in terms of R, α , h, g.)

a) Assuming $h > R$, what is the speed at point B?

y) $ma_T = -mg$ $a_T = -g$

x) $\frac{mv^2}{R} = N$

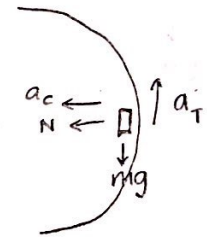
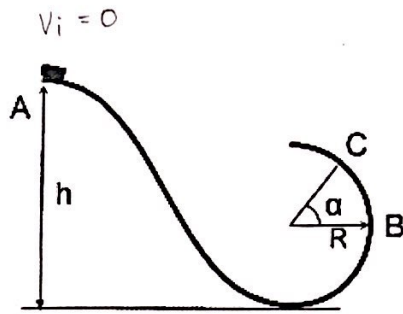
$v^2 = \frac{NR}{m}$

$E_f = E_i + W_{nc}$ no friction!

$\frac{1}{2}mv^2 + mgh = mgh$

$\frac{v^2}{2} + gR = gh$

$v = \sqrt{2gh - 2gR}$ ✓ (+15)

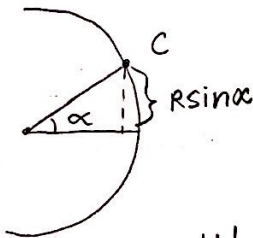


b) The block loses contact with the surface at point C, where the radius makes angle α with the horizontal. What is the height h of point A?

"loses contact" means $F_N = 0$

$mgh = \frac{1}{2}mv^2 + mgh$

$gh = \frac{1}{2}v^2 + gh$



y) $0 = -mg - N \sin \alpha$

$mg = -N \sin \alpha$

$N = \frac{-mg}{\sin \alpha}$

x) $\frac{mv^2}{R} = \frac{mg \sin \alpha}{N \cos \alpha}$

$\frac{mv^2}{R} = \frac{-mg \cos \alpha}{\sin \alpha}$

$v = \sqrt{\frac{-Rg \cos \alpha}{\sin \alpha}}$ (+11)

$E_f = E_i + W_{nc}$

$\frac{1}{2}mv^2 + mgh = mgh$

$\frac{1}{2}m \left[\frac{-Rg \cos \alpha}{\sin \alpha} \right] + mg(R + R \sin \alpha) = mgh$

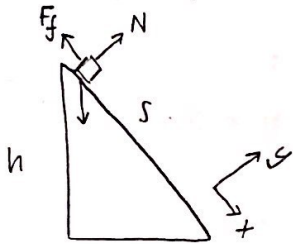
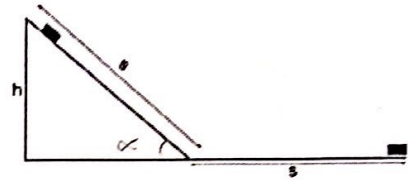
$\frac{-R \cos \alpha}{2 \sin \alpha} + R + R \sin \alpha = h$

$h = \frac{-R \cos \alpha}{2 \sin \alpha} + R(1 + \sin \alpha)$

$h = R \left(+ \frac{\sin \alpha}{2 \sin \alpha} + 1 + \sin \alpha \right)$

Problem 2

A body with zero initial velocity slides along an inclined plane forming an angle α with the horizon, then continues moving over a horizontal plane, and then comes to rest. The friction coefficient is the same on the inclined plane and on the horizontal surface. Find the coefficient of friction μ if the body covers the same distance S on the horizontal plane as on the inclined plane.



$$y) 0 = -mg \cos \theta + N$$

$$N = mg \cos \theta$$

$$x) ma = mg \sin \theta - F_f$$

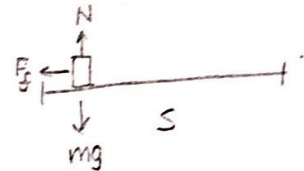
$$ma = mg \sin \theta - \mu N$$

$$ma = mg \sin \theta - \mu mg \cos \theta$$

$$a = g \sin \theta - \mu g \cos \theta$$

$$V_f^2 = V_i^2 + 2a \Delta x$$

$$V_f^2 = 0 + 2(g \sin \theta - \mu g \cos \theta) S$$



$$x) ma = -F_f$$

$$ma = -\mu mg$$

$$a = -\mu g$$

$$V_f^2 = V_i^2 + 2a \Delta x$$

$$0 = V_i^2 + 2(-\mu g) S$$

$$2Sg(\sin \theta - \mu \cos \theta) = 2\mu Sg$$

$$\sin \theta - \mu \cos \theta = \mu$$

$$\sin \theta = \mu + \mu \cos \theta$$

$$\sin \theta = \mu(1 + \cos \theta)$$

$$\mu = \frac{\sin \theta}{1 + \cos \theta}$$

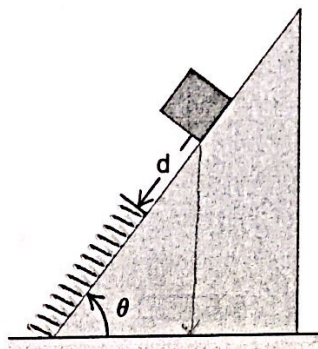
✓ (+30)

20/30

Problem 3

$V_i = 0$

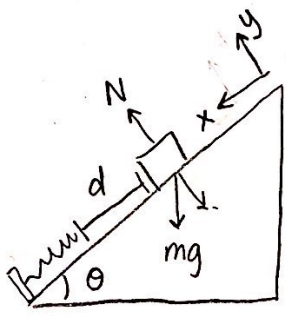
A block of mass $m=1$ kg slides without friction down an inclined plane making angle $\theta=30^\circ$ with the horizontal, lands on a spring with the spring constant $k=10$ N/m, and bounces back. The initial distance between the block and the end of the undeformed spring is $d=1$ m. Find the maximal speed the block will reach.



$\theta = 30^\circ$ $m = 1$ kg
 $k = 10$ N/m

$V_{max} = ?$

• V_{max} will occur when spring is no longer compressed



~~$E_f = E_i + W^{nc}$~~
 ~~$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + Fd\cos\theta + mgh$~~
 ~~$\frac{1}{2}m\boxed{v^2} = \frac{1}{2}kx^2 + Fd\cos\theta + mg(d\sin\theta + x\sin\theta)$~~
 ~~$mv^2 = kx^2 + 2Fd\cos\theta + 2mg(d\sin\theta + x\sin\theta)$~~
 ~~$mv^2 = 10x^2 + 9.81 + 19.62(0.5 + \frac{x}{2})$~~
 ~~$v = 8.831$ m/s~~

$W = Fd\cos\theta$
 $W = mg\sin\theta d$

no friction!
 ~~$E_i + W^{nc} = E_f$~~

top of incline bottom, spring fully compressed

$mg(d\sin\theta + x\sin\theta) = \frac{1}{2}kx^2$

$4.905 + 9.81\sin\theta x = 5x^2$

$0 = 5x^2 - 9.81\sin\theta x - 4.905$

$x = 1.596$ m

initially, when spring is fully compressed

immediately after losing contact with spring

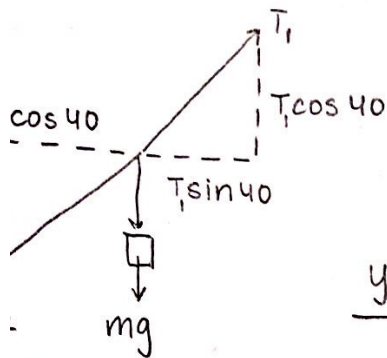
$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$v^2 = 10(1.596)^2$

$V = 5.047$ m/s

Problem 4

If mass $M=10$ kg is attached as shown in the figure, what is the tension in string 2?



$$m = 10 \text{ kg}$$

$$\text{y)} \quad ma_y = T_1 \cos 40 - T_2 \sin 40 + mg$$

$$0 = T_1 \cos 40 - T_2 \sin 40 + mg \quad \times + 25$$

$$\text{x)} \quad 0 = T_1 \sin 40 - T_2 \cos 40 \quad \times$$

$$T_1 = \frac{T_2 \cos 40}{\sin 40}$$

just mixed up sines + cosines

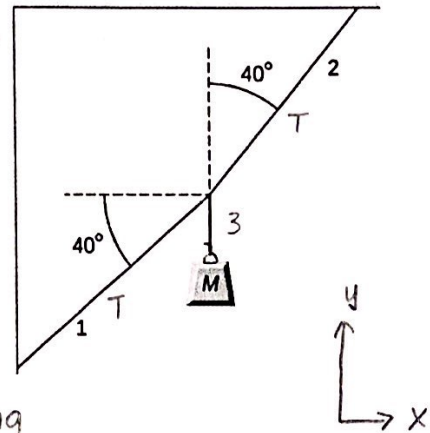
$$0 = \frac{T_2 \cos^2 40}{\sin 40} - T_2 \sin 40 - mg$$

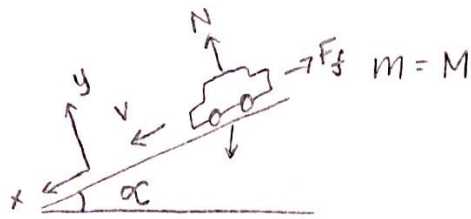
$$mg = T_2 \left(\frac{\cos^2 40}{\sin 40} - \sin 40 \right)$$

$$T_2 = \frac{mg}{\left(\frac{\cos^2 40}{\sin 40} - \sin 40 \right)} = 363.133 \text{ N}$$

$$\left(\frac{\cos^2 40}{\sin 40} - \sin 40 \right)$$

363 N





Problem 5

A car with mass M runs downhill with its engine turned off at a constant speed v . The road makes angle α with the horizontal.

- a) Find the coefficient of friction.

$$\mu = ?$$

$$\begin{aligned} \text{y)} \quad 0 &= -mg \cos \alpha + N \\ N &= mg \cos \alpha \end{aligned}$$

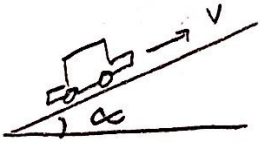
$$\text{x)} \quad 0 = mg \sin \alpha - \mu N$$

$$0 = mg \sin \alpha - \mu mg \cos \alpha$$

$$\mu \cos \alpha = \sin \alpha$$

$$\boxed{\mu = \tan \alpha}$$

- b) What is the required power P to make this car run upward at the same angle with the same speed v ?



$$\begin{aligned} P &= \vec{F} \cdot \vec{v} \quad \text{or} \quad P = \frac{W}{\Delta t} \\ \vec{a} &= 0, \text{ so } +3 \\ \vec{F} &\text{ is } 0 \end{aligned}$$

$$W = \Delta KE$$

$$W = \frac{1}{2} m v^2 - \frac{1}{2} m v^2$$

$$v \text{ is const, so } W = 0$$

$$\boxed{P = 0 \text{ W}}$$