Midterm 2

Physics 1A (Lec 3) 2019

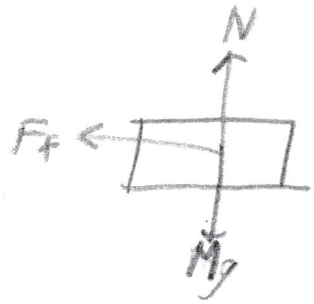
Time to complete the exam: 90 min

Each problem is worth 20 points. If a problem has parts (a) and (b), they are 10 points each. It is not sufficient to present the final answer. You need to show the solution and justify your steps at the level of detail that would be sufficient for your fellow classmate (or grader) to understand how you arrived at the final answer. Please write your solutions in the spaces below each question. You can use the back sides of the pages as scrap paper. Numerical answers need not have more significant figures than the numbers provided in the problem.

1	2	3	4	5	6	total
20	20	16	20	20	20	116

20

Problem 1 A bullet with mass m=10 g moves horizontally with speed v_0 = 270 m/s as it hits a box with mass M=0.5 kg and comes out on the other side of the box with speed v_1=90 m/s. The coefficient of friction between the box and the floor is μ=0.3. Find the distance by which the box will move after being hit by the bullet.



$mv_i = mv_f$

$mv_0 = Mv + mv_1$

$0.01(270) = 0.5(v) + 0.01(90)$

$v = 3.6\ m/s$

$N = Mg$

$Ma = F_f = \mu N$

$Ma = \mu Mg$

$a = \mu g$

$v_f = v_0 + at$

$0 = 3.6 - \mu g t$

$t = \frac{3.6}{\mu g} = \frac{3.6}{0.3(9.8)}$

$t = 1.22\ s$

$x = v_0 t + \frac{1}{2}at^2$

$x = 3.6(1.22) - \frac{1}{2}(0.3)(9.8)(1.22)^2$

$x = \boxed{2.204\ m}$

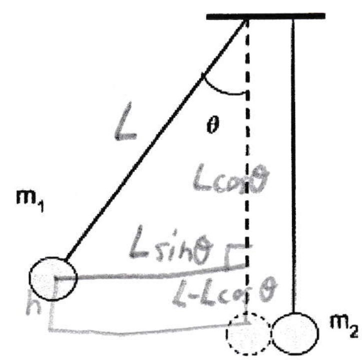
$\cancel{\frac{1}{2}Mv^2 = \mu Mgd}$

$\cancel{\frac{1}{2}v^2 = \mu gd}$

$\cancel{d = \frac{v^2}{2\mu g}}$

$\cancel{d = 2.204\ m}$

Problem 2. Two balls with masses m_1 and m_2 ($m_1 < m_2$) are suspended as shown. Each thread has length L. The ball m_1 is pulled away to an angle θ with the vertical and then released. Express all answers using the symbolic quantities L, m_1, m_2, θ, g.



a) What is the velocity of this ball before the impact?

$$mgh = \frac{1}{2}mv^2$$

$$gh = \frac{1}{2}v^2 \qquad h = L - L\cos\theta$$

$$g(L - L\cos\theta) = \frac{1}{2}v^2$$

$$v = \boxed{\sqrt{2gL(1-\cos\theta)}}$$

10

b) What is the velocity of the balls immediately after an inelastic collision (when the two of them stick together)?

$$m_1 v_0 = (m_1 + m_2)v_f$$

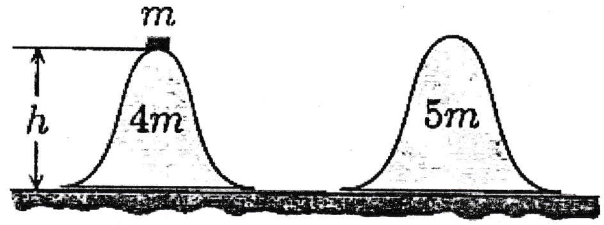
$$m_1\sqrt{2gL(1-\cos\theta)} = (m_1 + m_2)v_f$$

$$V_f = \boxed{\frac{m_1\sqrt{2gL(1-\cos\theta)}}{m_1 + m_2}}$$

10

16

Problem 3. Two bell-shaped "hills" with masses *4 m* and *5 m*, respectively, can move without friction on a smooth surface of the table. The height of each hill is *h*. A small coin with mass m is placed on top of one hill. Express all answers in terms of *h, m, g,* etc.



10 a) If the coin starts with zero speed at the top, what is its speed when it reaches the surface of the table? *[Hint: it is important that the "hill" itself moves as the coin slides down.]*

$4mv_f = mv$ $\quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}(4m)v_f^2$

$4v_f = v$ $\quad gh = \frac{1}{2}v^2 + 2v_f^2$

$v_f = \frac{v}{4}$ $\quad gh = \frac{1}{2}v^2 + 2\left(\frac{1}{4}\right)^2 v^2$

$gh = \frac{5}{8}v^2$

$v = \boxed{\sqrt{\frac{8gh}{5}}}$

6 b) What is the maximal height to which the coin can climb the second hill? *[Hint: it is important that the "hill" itself moves as the coin slides up.]*

$mv = (5m + m)v_f$ $\quad \frac{1}{2}mv^2 = \frac{1}{2}(\overset{6}{\cancel{4}}m)v_f^2 + mgh_f$

$mv = 6mv_f$ $\quad \frac{1}{2}v^2 = 2v_f^2 + gh_f$

$v = 6v_f$ $\quad \frac{1}{2}\left(\sqrt{\frac{8gh}{5}}\right)^2 = 2\left(\frac{\sqrt{\frac{8gh}{5}}}{6}\right)^2 + gh_f$

$v_f = \frac{v}{6}$

$\frac{4gh}{5} = \frac{4gh}{45} + gh_f$

$\boxed{\frac{32h}{45}} = h_f$

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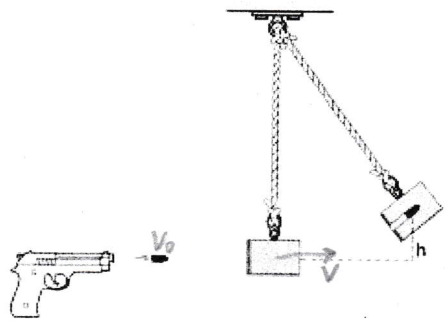
Problem 4. A bullet with mass m=*10 g* is fired horizontally into a wooden block of mass M=*100 g* suspended by a long cord. After the impact, the bullet is stucks in the block, and the block swings to height h=*10 cm* above its initial position.



a) What was the bullet's speed before the impact?

$$\frac{1}{2}(m+M)v^2 = (m+M)gh$$

$$\frac{1}{2}v^2 = gh$$

$$v = \sqrt{2gh}$$

$$mv_0 = (m+M)v = (m+M)\sqrt{2gh}$$

$$0.01\,v_0 = (0.01+0.1)\sqrt{2(9.8)(0.1)}$$

$$v_0 = \boxed{15.4\ m/s}$$

b) What amount of energy was converted into non-mechanical forms of energy?

$$\frac{1}{2}mv_0^2 = \frac{1}{2}(m+M)v^2 + E = \frac{1}{2}(m+M)(\sqrt{2gh})^2 + E$$

$$\frac{1}{2}(0.01)(15.4)^2 = \frac{1}{2}(0.01+0.1)(\sqrt{2(9.8)(0.1)})^2 + E$$

$$E = \boxed{1.078\ J}$$

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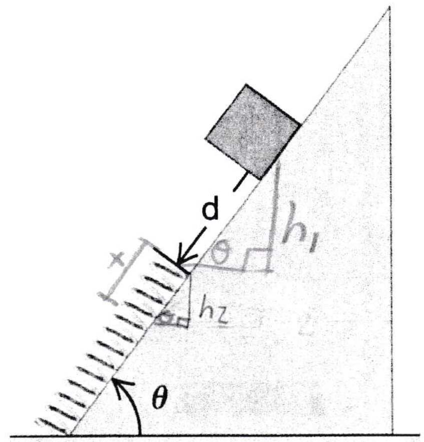
Problem 5. A block of mass m slides down an inclined plane making angle θ with the horizontal, lands on a spring with the spring constant k, and bounces back. The initial distance between the block and the end of the undeformed spring is d. Find the length x by which the spring is compressed when the block is at the lowest height. Neglect friction. Express the answer in terms of m,d,k,g,θ



$$mgh = \frac{1}{2}kx^2$$

$$h = h_1 + h_2$$

$$mgh_1 = \frac{1}{2}\cancel{m}v^2$$

$$\frac{1}{2}mv^2 + mgh_2 = \frac{1}{2}kx^2$$

$$h_1 = d\sin\theta$$

don't actually need to do this

$$mgd\sin\theta = \frac{1}{2}mv^2$$

$$gd\sin\theta = \frac{1}{2}v^2$$

$$v = \sqrt{2gd\sin\theta}$$

$$h_2 = x\sin\theta$$

$$\frac{1}{2}mv^2 + mgx\sin\theta = \frac{1}{2}kx^2$$

but it still works

$$\frac{1}{2}m\left(\sqrt{2gd\sin\theta}\right)^2 + mgx\sin\theta = \frac{1}{2}kx^2$$

this is just PE;

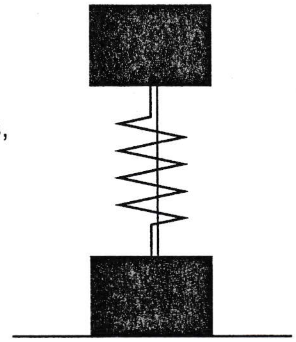
$$\frac{1}{2}kx^2 - mgx\sin\theta - \frac{1}{2}m(2gd\sin\theta) = 0$$

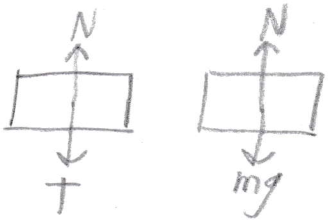
$$2kx^2 - 2mgx\sin\theta - 2mgd\sin\theta = 0$$

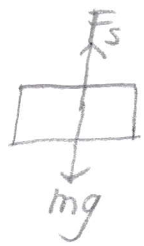
$$x = \frac{2mg\sin\theta + \sqrt{(2mg\sin\theta)^2 - 4(k)(-2mgd\sin\theta)}}{2k}$$

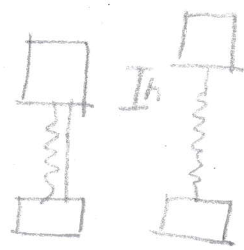
$$\boxed{x = \frac{2mg\sin\theta + \sqrt{4m^2g^2\sin^2\theta + 8kmgd\sin\theta}}{2k}}$$

Problem 6. Two identical boxes of mass *m* are connected by a compressed massless spring and a massless thread that keeps the spring from expanding. The spring constant is *k*, and it is initially compressed by distance *x* as compared to its uncompressed length. When the thread breaks, the top box jumps up and pulls up the lower box. What is the minimal value of *x* for which the lower box can jump up and separate from the ground? (Express the answer in terms of *m*,*g*,*k*.)









$\frac{1}{2}kx^2 = \frac{1}{2}mv_i^2$

$\frac{1}{2}mv_i^2 = mgh + \frac{1}{2}k(h-x)^2$

$\frac{1}{2}kx^2 = mgh + \frac{1}{2}k(h-x)^2$

$F_s = k(h-x)$

$k(h-x) = mg$

$h - x = \frac{mg}{k}$

$h = \frac{mg}{k} + x$

$\frac{1}{2}kx^2 = mg\left(\frac{mg}{k} + x\right) + \frac{1}{2}k\left(\frac{mg}{k}\right)^2$

$\frac{1}{2}kx^2 = \frac{m^2g^2}{k} + mgx + \frac{1}{2}\frac{m^2g^2}{k}$

$kx^2 - 2mgx - 3\frac{m^2g^2}{k} = 0$

$k^2x^2 - 2kmgx - 3m^2g^2 = 0$

$x = \frac{2kmg + \sqrt{(2kmg)^2 - 4(k^2)(-3m^2g^2)}}{2k^2}$

$x = \frac{2kmg + \sqrt{4k^2m^2g^2 + 12k^2m^2g^2}}{2k^2}$

$x = \frac{2kmg + 4kmg}{2k^2}$

$\boxed{x = \frac{3mg}{k}}$

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