Midterm 1

Physics 1A (Lec 3) 2020

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ID number:

Time to complete the exam: 90 min

Each problem is worth 30 points. If a problem has parts (a) and (b), they are 15 points each. If a problem has parts (a), (b), and (c), they are 10 points each. It is not sufficient to present the final answer. You need to show the solution and justify your steps at the level of detail that would be sufficient for your fellow classmate (or grader) to understand how you arrived at the final answer. Please write your solutions in the spaces below each question. You can use the back sides of the pages as scrap paper. Numerical answers need not have more significant figures than the numbers provided in the problem.

1	2	3	4	5	total

Problem 1

A body falls vertically from the height $h = 44\ m$ with zero initial velocity. Neglect air resistance.

a) What distance will be traveled during the first second?

→ General Equation: $y(t) = y_0 + V_0 t + \frac{1}{2}at^2$

→ for us, $y_0 = h$, $V_0 = 0$, $a = -g$

→ $y(t) = h - \frac{1}{2}gt^2$

$$\Delta y \equiv |y(1\ sec) - y(0\ sec)| = \left|h - \frac{1}{2}g(1\ sec)^2 - \left(h - \frac{1}{2}g(0\ sec)^2\right)\right|$$

$$= \frac{1}{2}g(1\ sec)^2$$

$$= \frac{1}{2}(9.8\ m/s^2)(1\ sec)^2 = \boxed{4.9\ m}$$

b) What distance will be traveled during the last second?

We need to find when object hits the ground.

$$y(t_{impact}) = h - \frac{1}{2}g\,t_{impact}^2 \stackrel{!}{=} 0 \rightarrow t_{impact} = \sqrt{\frac{2h}{g}}$$

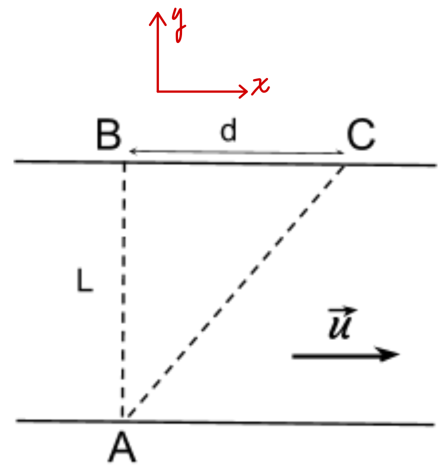
$$\Delta y = |\underbrace{y(t_{impact})}_{0} - y(t_{impact} - 1)|$$

$$= |y(t_{impact} - 1)| = h - \frac{1}{2}g\left(\sqrt{\frac{2h}{g}} - 1\ sec\right)^2$$

$$= 44m - \frac{1}{2}(9.8\ m/s^2)\left(\sqrt{\frac{2(44\ m)}{9.8\ m/s^2}} - 1\ sec\right)^2$$

$$= \boxed{24.47\ m}$$

Problem 2



Linda and Michael swim across the river starting at point A. They can swim with the same speed V with respect to the water. Linda swims along the straight line AB, while Michael swims with a velocity perpendicular to the river and lets the flow take him to point C some distance d=240 m downstream. The time it took Linda to swim from A to B was t_{AB}=25 min. The time it took Michael to swim from A to C was t_{AC}=20 min.

a) What is the speed of the river u?

→ Michael's velocity vector with respect to the ground is given by: $\vec{V}_{M/G} = \vec{V}_{M/R} + \vec{V}_{R/G}$

→ $\vec{V}_{M/R} = V\hat{y}$, $\vec{V}_{R/G} = u\hat{x} \Rightarrow \vec{V}_{M/G} = u\hat{x} + V\hat{y}$

→ Michael's Equation of Motion in x-direction is then, $x(t) = ut$

→ $x(t_{AC}) = d = u\, t_{AC} \rightarrow u = \frac{d}{t_{AC}} \rightarrow \boxed{u = \frac{(240\text{ m})}{(20\text{ min})} \cdot \frac{1\text{ min}}{60\text{ sec}} = \frac{1}{5}\text{ m/s} = 0.2\text{ m/s}}$

b) What is the width of the river L?

→ Similarly, $V = \frac{L}{t_{AC}} \rightarrow L = V \cdot t_{AC}$ (1)

→ Need to eliminate V

→ For Linda: $\vec{V}_{L/G} = \vec{V}_{L/R} + \vec{V}_{R/G} \rightarrow \vec{V}_{L/R} = \vec{V}_{L/G} - \vec{V}_{R/G}$

→ $\vec{V}_{L/G} = V_{L/G}\hat{y}$, $\vec{V}_{L/R} = ?$, $\vec{V}_{R/G} = u\hat{x}$

→ Note: We don't know exact direction of $\vec{V}_{L/R}$ but we do know that $|\vec{V}_{L/R}| = V$

$|\vec{V}_{L/R}| = |\vec{V}_{L/G} - u\hat{x}| \Rightarrow V^2 = V_{L/G}^2 + u^2$ (★)

→ Now note that $V_{L/G} \cdot t_{AB} = L$ (2)

→ From (1) & (2)

$$V_{L/G} \cdot t_{AB} = V t_{AC} \rightarrow V_{L/G} = \frac{t_{AC}}{t_{AB}} V$$

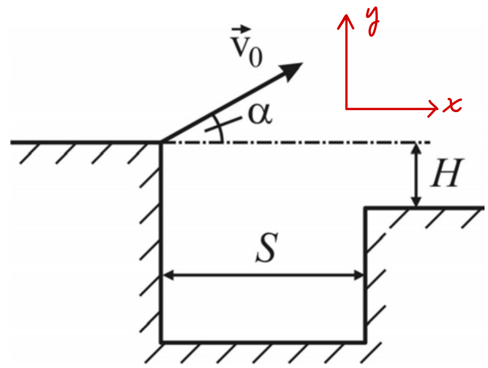
→ Combine this w/ ★

$$\rightarrow V^2 = \left(\frac{t_{AC}}{t_{AB}}\right)^2 V^2 + u^2 \rightarrow V^2 = \frac{u^2}{\left(1 - \left(\frac{t_{AC}}{t_{AB}}\right)^2\right)}$$

→ From (1) $\boxed{L = t_{AC} \cdot \frac{u}{\left(1 - \left(\frac{t_{AC}}{t_{AB}}\right)^2\right)^{1/2}} = 400\text{ m}}$

Problem 3

An arrow is shot across a river, which is 500-meters wide (S=500 m). The left bank is H=15 m higher than the right bank. If the initial velocity is at the angle α=45 degrees to the horizontal, what is the minimal speed for which the arrow does not fall into the river, but lands on the opposite bank?



→ We begin with the equations of motion:

$$\begin{cases} y(t) = H + V_0 \sin\alpha\, t - \frac{1}{2} g t^2 \\ x(t) = V_0 \cos\alpha\, t \end{cases}$$

→ Looking for V_0 so that $x(t_{impact}) = S$

→ To do this, we will eliminate t in the y equation in favor of x

→ $t = \frac{x}{V_0 \cos\alpha}$

→ $y(x(t)) = H + V_0 \sin\alpha \left(\frac{x}{V_0 \cos\alpha}\right) - \frac{1}{2} g \left(\frac{x}{V_0 \cos\alpha}\right)^2$

$= H + x \tan\alpha - \frac{1}{2} g \left(\frac{x}{V_0 \cos\alpha}\right)^2$

→ We are interested in the time when $x = S$ and $y = 0$. So we replace $x \to S$, $y \to 0$ and solve for V_0

→ $0 = H + x\tan\alpha - \frac{1}{2} g \left(\frac{S}{V_0 \cos\alpha}\right)^2$

→ $\frac{2}{g}(H + S\tan\alpha) = \frac{S^2}{V_0^2 \cos^2\alpha}$ → $\frac{V_0^2 \cos^2\alpha}{S^2} = \frac{g}{2(H + S\tan\alpha)}$ → $\boxed{V_0 = \sqrt{\frac{g S^2}{2\cos^2\alpha (H + S\tan\alpha)}}}$

→ $\boxed{V_0 = \sqrt{\frac{(9.8\,m/s^2)\cdot(500\,m)^2}{2\cos^2(\frac{\pi}{4})\cdot(15\,m + 500\,m \cdot \tan(\pi/4))}} = 68.97\ m/s}$

Problem 4

An object moves around a circle of radius R=2 m with the angular velocity given by the equation $\omega = \pi\, b\, t$, where $b = 2\, s^{-2}$, $\pi = 3.14$, and t is time.

(a) What is the total acceleration at time t=1 s?

$\rightarrow \omega = \dot{\theta} = \pi b t, \quad \alpha = \ddot{\theta} = \pi b \rightarrow a_T = R\alpha = R\pi b = (2\,m)\pi(2\,s^{-2}) = 4\pi\ m/s^2$

→ The Centripital acceleration must be included too:

$a_c = \frac{v^2}{R} = \omega^2 R = \pi^2 b^2 t^2 R$

$\rightarrow |\vec{a}_{tot}| = \sqrt{a_c^2 + a_T^2} = \sqrt{(\pi^2 b^2 t^2 R)^2 + (R\pi b)^2}$

$\rightarrow |\vec{a}_{tot}| = \sqrt{(\pi^2 (2\,sec^{-2})^2 (1\,sec)^2 (2\,m))^2 + ((2\,m)\pi(2\,sec^{-2}))^2} = \sqrt{(8\pi^2\ m/sec^2)^2 + (4\pi\ m/sec^2)^2} = 79.95\ m/s^2$

(b) What is the distance traveled from t=0 to t=1 s?

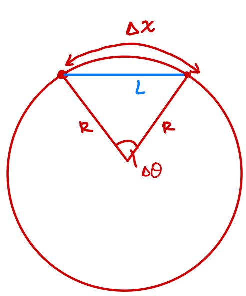
$\omega = \dot{\theta} = \pi b t$; Integrate on both sides

$\rightarrow \int_0^t \frac{d\theta}{dt}\, dt = \int_0^t \pi b t\, dt = \theta(t) - \cancel{\theta(0)} = \frac{\pi b}{2} t^2$

$\rightarrow x(t) = R\theta(t) = \frac{R\pi b}{2} t^2$

$\Delta x = x(1\,sec) - x(0\,sec) = \frac{R\pi b}{2}(1\,sec)^2 = \frac{(2\,m)\pi}{2}(2\,sec^{-2})(1\,sec)^2 = \boxed{2\pi\ m}$

(c) What is the displacement from t=0 to t =1 s ?



Want to find L

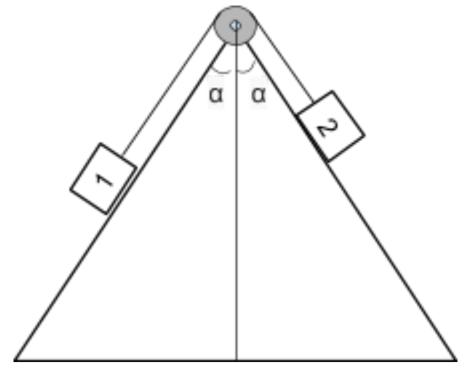
$\Delta\theta = \theta(1\,sec) - \theta(0\,sec) = \frac{\pi b}{2}(1\,sec)^2$

$L = 2R \sin\left(\frac{\Delta\theta}{2}\right)$

$= 2R \sin\left(\frac{\pi b}{4}(1\,sec)^2\right)$

$\rightarrow L = 2(2\,m)\cdot \sin\left(\frac{\pi(2\,sec^{-2})}{4}\cdot(1\,sec)^2\right) = \boxed{4\,m}$

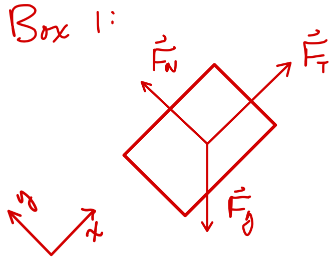
Problem 5



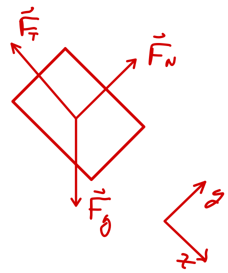
Two boxes with masses m_1 and m_2 are connected by a massless rope over a massless frictionless pulley, as shown.
The angle α. Neglect friction. Express the answers in terms of m_1, m_2 and α.

a) What is the acceleration of the box 1?

Box 1:



Box 2



Box 1 : $\hat{x}$: $F_T - m_1 g \cos\alpha = m_1 a_x$ (1)

$\hat{y}$: $-m_1 g \sin\alpha + F_N = 0$

Box 2 : $\hat{x}$: $-F_T + m_2 g \cos\alpha = m_2 a_x$ (2)

$\hat{y}$: $F_N - m_2 g \sin\alpha = 0$

→ add equations (1) & (2)

$$(m_2 - m_1) g \cos\alpha = (m_1 + m_2) a_x \rightarrow \boxed{a_x = \frac{(m_2 - m_1)}{(m_2 + m_1)} g \cos\alpha}$$

b) What is the force of tension in the rope connecting the boxes?

From (1) in the previous part

$$\rightarrow F_T = m_1 g \cos\alpha + m_1 a_x = m_1 g \cos\alpha + m_1 \frac{(m_2 - m_1)}{(m_2 + m_1)} g \cos\alpha$$

$$\rightarrow \boxed{F_T = \frac{2 g m_1 m_2 \cos\alpha}{(m_1 + m_2)}}$$