Midterm 1

Physics 1A (Lec 5) 2019

Name

ID numb

Time to complete the exam: 90 min

Each problem is worth 20 points. If a problem has parts (a) and (b), they are 10 points each. It is not sufficient to present the final answer. You need to show the solution and justify your steps at the level of detail that would be sufficient for your fellow classmate (or grader) to understand how you arrived at the final answer. Please write your solutions in the spaces below each question. You can use the back sides of the pages as scrap paper. Numerical answers need not have more significant figures than the numbers provided in the problem.

1	2	3	4	5	6	total
20	15	20	20	17	20	112

20

Problem 1

Between t=0 and $t=t_1$, a rocket moves straight upward with an speed given by $v(t)=A - B\,t^2$, where A and B are constants. The initial position at t=0 is zero.

(a) What is the rocket's acceleration as a function of time?

$$a(t) = \frac{d}{dt} v(t)$$
$$= -2Bt$$

(b) What is the rocket's position as a function of time?

$$x(t) = \int v(t)\, dt$$
$$= At - \frac{Bt^3}{3} + C$$

when $t = 0$,

$$x(0) = 0 \Rightarrow A \times 0 - \frac{B \times 0^3}{3} + C = 0 \Rightarrow C = 0$$

Therefore $x(t) = At - \frac{Bt^3}{3}$

$\Delta t + \Delta t + \Delta t + \Delta t$

Problem 2 Drops of water fall at regular intervals Δt from the roof of a building of height H=16m, and the first drop strikes the ground at the same moment as the fifth drop detaches from the roof.

(a) Find the time interval Δt between the drops.

- the time interval between the 1st drop and 5th drop

$$t_{15} = 4\Delta t \quad \checkmark$$

- the time 1st drop takes to strike ground

$$t = t_{15} = 4\Delta t$$

y: $x(t) = x_0 + v_0 t + \frac{at^2}{2}$ $\qquad x(t) = 0m$

$0m = 16m + 0m/s \cdot t - \frac{g}{2}t^2$ $\qquad x_0 = H = 16m$

$\qquad v_0 = 0m/s$

$$\frac{g}{2}t^2 = 16m$$

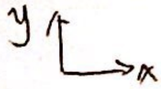
$$t = \sqrt{\frac{32}{g}}\,s$$

+7

$\because t = 4\Delta t \qquad \therefore \Delta t = \frac{t}{4} = \frac{8}{g}s = 0.8s$ X

(b) Find the height of the second drop above the ground when the first drop hits the ground.

$$t_{2nd\ in\ air} = t_{1st\ strike\ on\ ground} - t_{12}$$



$$= 4\Delta t - \Delta t$$

$$= 3\Delta t = 2.4s$$

+8

$$x(t) = x_0 + v_0 t + \frac{at^2}{2}$$

displacement of 2nd drop: $x(t_{2nd\ in\ air}) = 16m - \frac{g(3\Delta t)^2}{2}$

$$= 16 - \frac{g}{2}(2.4)^2$$

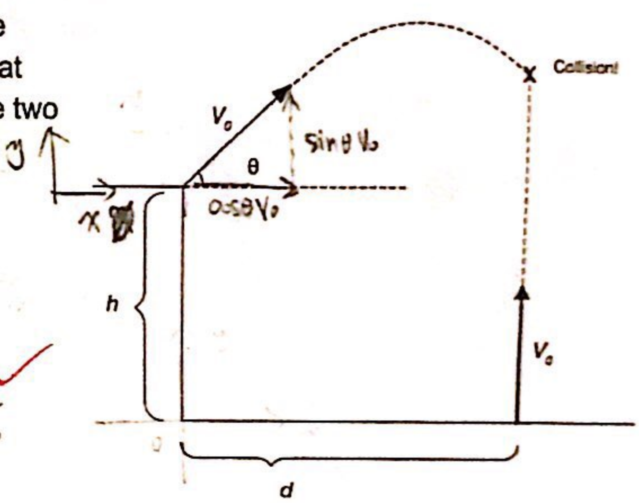
$$= 16 - \frac{9.8}{2} \times 9 \times (0.8)^2$$

$$= -12.2m$$

X

x above the ground $= 16 - 12.2 = 3.8m$

Problem 3 Two identical rockets collide after they were launched at the same time with the same initial speed v_0. One of the rockets was launched from the ground straight up, the other one was launched at angle θ from a roof of a building at height h above ground. The horizontal distance between the two launch points is d. How high above the ground will the two rockets collide?



Let t be time when two rockets collide

x: $\cos\theta V_0 t = d \Rightarrow t = \dfrac{d}{\cos\theta V_0}$

y: displacement of rocket on the roof:

$$\Delta S_r = 0 + \sin\theta V_0 t - \frac{g}{2}t^2$$

displacement of rocket on the ground

$$\Delta S_g = 0 + V_0 t - \frac{g}{2}t^2$$

$$h + \left(0 + \sin\theta V_0 t - \frac{g}{2}t^2\right) = 0 + V_0 t - \frac{g}{2}t^2$$

$$h + \sin\theta V_0 t - \cancel{\frac{g}{2}t^2} = V_0 t - \cancel{\frac{g}{2}t^2}$$

$$h + (\sin\theta - 1)V_0 t = 0$$

$$t = \frac{-h}{(\sin\theta - 1)V_0} = \frac{h}{(1-\sin\theta)V_0}$$

$$\Delta S_g = 0 + V_0 t - \frac{g}{2}t^2$$

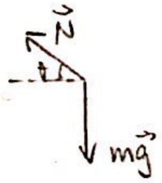
$$= \frac{V_0 h}{(1-\sin\theta)V_0} - \frac{g}{2}\frac{h^2}{(1-\sin\theta)^2 V_0^2}$$

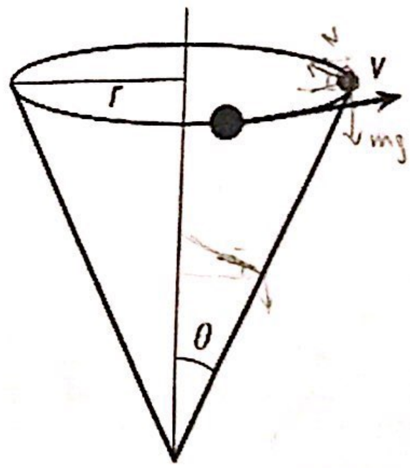
$$= \frac{h}{1-\sin\theta} - \frac{h^2 g}{2(1-\sin\theta)^2 V_0^2}$$

Problem 4

A bead rolls on the inside of a cone with apex half-angle equal θ (see Fig.). The cone is positioned vertically, the bead is making a circle of radius r, and the force of friction is zero. What is the speed of the bead?

$x \leftarrow \uparrow y$





x: $ma_c = \cos\theta N$

y: $ma = \sin\theta N - mg$

$0 = \sin\theta N - mg$

$\sin\theta N = mg$

$N = \dfrac{mg}{\sin\theta}$

$a_c = \dfrac{v^2}{r}$

$m\dfrac{v^2}{r} = \cos\theta N$

$m\dfrac{v^2}{r} = \dfrac{\cos\theta}{\sin\theta} mg$

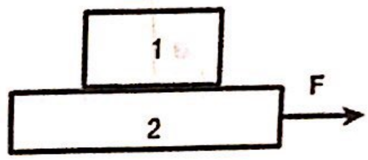
$v^2 = \dfrac{\cos\theta}{\sin\theta} gr$

$v = \sqrt{\dfrac{\cos\theta}{\sin\theta} gr}$

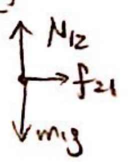
17

Problem 5

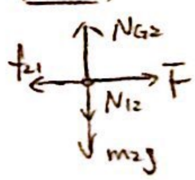
A box with mass m_1 is placed on top of another box with mass m_2, which rests on a horizontal surface. There is no friction between the bottom box and the surface. The coefficient of static friction between the two boxes is μ. Assume the kinetic friction coefficient is the same as the static friction coefficient. What is the maximal value of force F applied to the bottom box for which the two boxes move together without the top box sliding?



box 1:



box 2



box 1: y: $ma = N_{12} - m_1 g$

$0 = N_{12} - m_1 g$

$N_{12} = m_1 g$

x: $m_1 a_1 = f_{21}$

$f_{21} = \mu N_{12} = \mu m_1 g$ $\Rightarrow a_1 = \mu g$

box 1&2: x: $(\cancel{m_1+m_2}) a_2 = F - f_{21}$

$m_2 \quad = F - \mu m_1 g$ $\Rightarrow a_2 = \frac{F - \mu m_1 g}{m_1 + m_2}$

-3

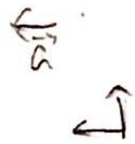
If move together

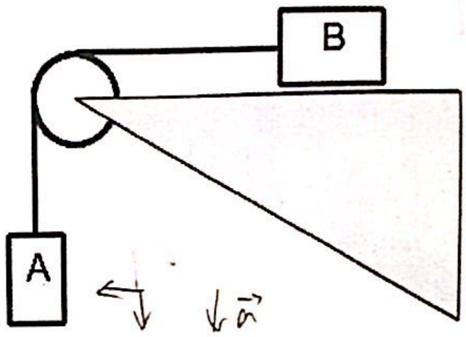
$a_1 = a_2$

$\mu g = \frac{F - \mu m_1 g}{m_1 + m_2}$

$F - \mu m_1 g = \mu g(m_1 + m_2)$

$F = \mu(2m_1 + m_2) g$





Problem 6

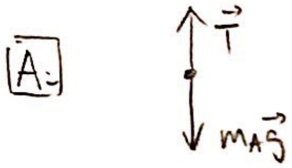
Blocks A and B have masses m_A and m_B, respectively. The coefficient of friction between B and the horizontal surface is μ.

(a) What is the acceleration with which A is moving?

$a_A = a_B$

A: $m_A a_A = -T + m_A g$

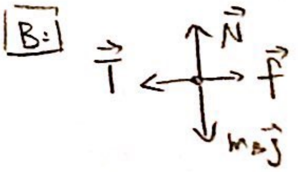
$T = m_A g - m_A a_A$ (1)



B: y: $m_B a_B = N - m_B g$

$0 = N - m_B g$

$N = m_B g$.



x: $m_B a_B = T - f$

$= T - \mu N = T - \mu m_B g$

$T = m_B a_B + \mu m_B g$ (2)

With (1) & (2) $m_A g - m_A a_A = m_B a_B + \mu m_B g$

Since $a_A = a_B$ $(m_A + m_B) a_A = m_A g - \mu m_B g \rightarrow a_A = \dfrac{m_A g - \mu m_B g}{m_A + m_B}$

(b) What is the tension in the thread?

From (a) $T = m_A g - m_A a_A$

$$a_A = \frac{m_A g - \mu m_B g}{m_A + m_B}$$

$$T = m_A g - (m_A)\frac{(m_A g - \mu m_B g)}{m_A + m_B}$$

$$= m_A g - \frac{m_A^2 g - \mu m_A m_B g}{m_A + m_B}.$$