Midterm 1

Physics 1A (Lec 3) 2019

Name: [illegible]

Each problem is worth 20 points. If a problem has parts (a) and (b), they are 10 points each. It is not sufficient to present the final answer. You need to show the solution and justify your steps at the level of detail that would be sufficient for your fellow classmate (or grader) to understand how you arrived at the final answer. Please write your solutions in the spaces below each question. You can use the back sides of the pages as scrap paper. Numerical answers need not have more significant figures than the numbers provided in the problem.

1	2	3	4	5	6	total
20	20	20	20	20	20	120

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Problem 1

Between $t=0$ and $t=t_1$, a rocket moves straight upward with an acceleration given by $a(t)=A-B\,t^2$, where A and B are constants. The initial position and velocity at $t=0$ are zero.

(a) What is the rocket's velocity as a function of time?

$v(t) = \int a(t)\,dt + C \rightarrow v = \int a(t)\,dt$

$v_0 = 0 = C$

$v = \int A - B t^2\,dt$

$\boxed{v(t) = At - \frac{B}{3}t^3}$

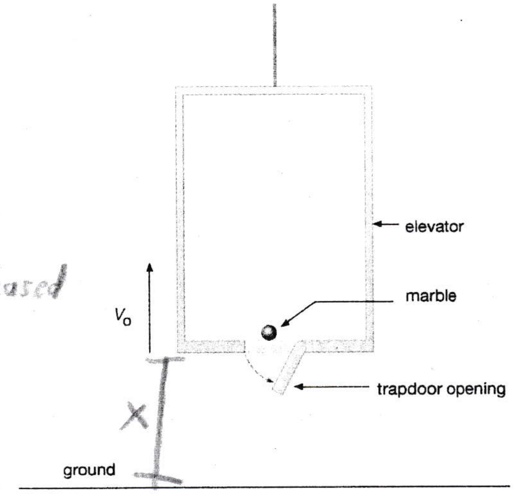
(b) What is the rocket's position as a function of time?

$x(t) = \int v(t) + C \rightarrow x(t) = \int At - \frac{B}{3}t^3\,dt$

$x_0 = 0 = C$

$\boxed{x(t) = \frac{A}{2}t^2 - \frac{B}{12}t^4}$

Problem 2



At time t=0, an elevator starts moving upward from the ground at a constant speed v_0 (which is not known). At a later time $t=T_1$, a trap door opens under a marble placed on the floor. The marble hits the ground at a still later time $t=T_2$. Express answers in terms of T_1, T_2, g.

(a) What is the speed v_0?

$$x = V_0 t + \tfrac{1}{2}at^2 \qquad T_{tot} = T_2 = T_1 + T_{released}$$

$$x = V_0 T_1 \qquad T_{released} = T_2 - T_1$$

$$-x = V_0(T_2 - T_1) - \tfrac{1}{2} g (T_2 - T_1)^2$$

$$0 = V_0 T_1 + V_0(T_2 - T_1) - \tfrac{1}{2} g (T_2 - T_1)^2 \qquad 0 = V_0 T_2 - \tfrac{1}{2} g (T_2 - T_1)^2$$

$$V_0 = \boxed{\frac{g(T_2 - T_1)^2}{2T_2}}$$

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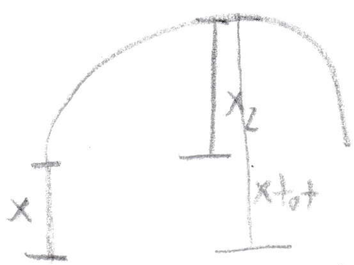
(b) What is the maximum height above the ground that the marble reaches?

t = time going up after release $\quad V_f = V_0 + at$

$$0 = V_0 - gt$$



$$t = \frac{V_0}{g}$$

$$X_2 = V_0 t + \tfrac{1}{2} a t^2$$

$$x = V_0 T_1$$

$$x = \frac{g T_1 (T_2 - T_1)^2}{2 T_2} \qquad X_2 = V_0\left(\frac{V_0}{g}\right) - \tfrac{1}{2} g \left(\frac{V_0}{g}\right)^2$$

$$X_{tot} = X + X_2 = \frac{g T_1 (T_2 - T_1)^2}{2 T_2} + \frac{V_0^2}{g} + \frac{V_0^2}{2g}$$

$$X_{tot} = \frac{g T_1 (T_2 - T_1)^2}{2 T_2} + \frac{V_0^2}{2g}$$

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$$X_{tot} = \boxed{\frac{g T_1 (T_2 - T_1)^2}{2 T_2} + \frac{g (T_2 - T_1)^4}{8 T_2^2}}$$

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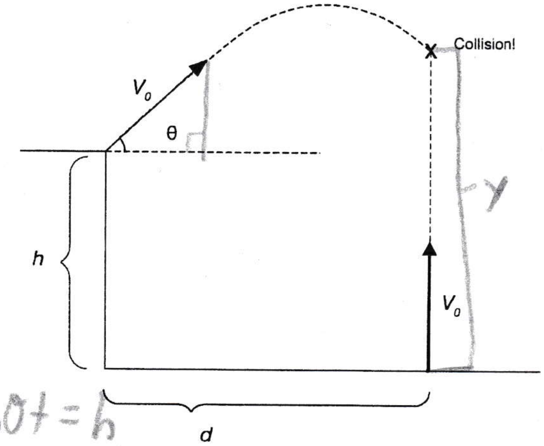
Problem 3 Two identical rockets collide after they were launched at the same time with the same initial speed v_0. One of the rockets was launched from the ground straight up, the other one was launched at angle θ from a roof of a building at height h above ground. The horizontal distance between the two launch points is d. What should the distance d be for the two rockets to collide?



$y = h + v_0 \sin\theta t - \frac{1}{2} g t^2$

$y = v_0 t - \frac{1}{2} g t^2$

$h + v_0 \sin\theta t - \frac{1}{2} g t^2 = v_0 t - \frac{1}{2} g t^2$

$h + v_0 \sin\theta t = v_0 t \quad v_0 t - v_0 \sin\theta t = h$

$t = \frac{h}{v_0 - v_0 \sin\theta}$

$d = v_0 \cos\theta\, t$

$d = v_0 \cos\theta \left(\frac{h}{v_0 - v_0 \sin\theta} \right)$

$\boxed{d = \frac{h \cos\theta}{1 - \sin\theta}}$

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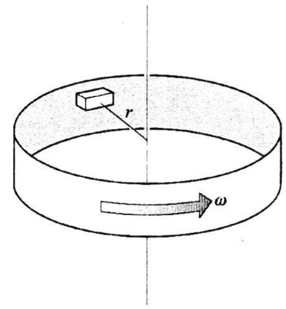
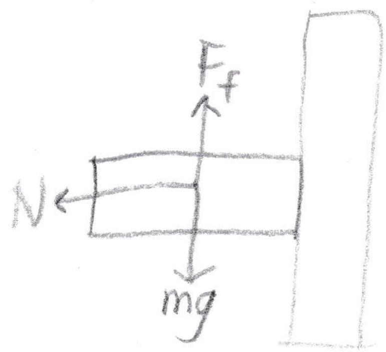
Problem 4
A wooden block co-rotates with a cylindrical shell of radius r. If the coefficient of static friction is μ, what is the lowest angular speed for which the block does not fall?





$N = F_c = m\frac{v^2}{r}$

$v = r\omega$

$F_f = \mu N = \mu m \frac{v^2}{r} = \mu m r \omega^2$

$0 = F_f - mg$

$0 = \mu m r \omega^2 - mg$

$0 = \mu r \omega^2 - g \qquad g = \mu r \omega^2$

$\boxed{\omega = \sqrt{\frac{g}{\mu r}}}$

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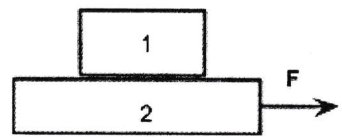
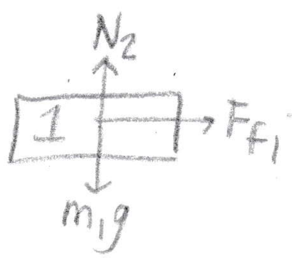
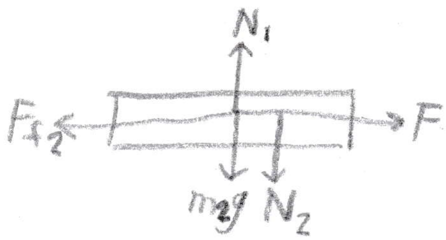
Problem 5

A box with mass m_1 is placed on top of another box with mass m_2, which rests on a horizontal surface. The coefficient of static friction between the two boxes is μ_1, the coefficient of friction between the box and the horizontal surface is μ_2, such that $\mu_1 > \mu_2$. Assume the kinetic friction coefficient is the same as the static friction coefficient. What is the maximal value of force F applied to the bottom box for which the two boxes move together without the top box sliding?







$N_2 = m_1 g$

$F_{f_1} = \mu_1 N_2 = \mu_1 m_1 g$

$m_1 a = \mu_1 m_1 g$

$a = \mu_1 g$

$m_2 a = F - F_{f_2}$

$F_{f_2} = \mu_2 N_1 + \mu_1 N_2$

$N_1 = m_2 g + N_2$

$N_1 = m_2 g + m_1 g$

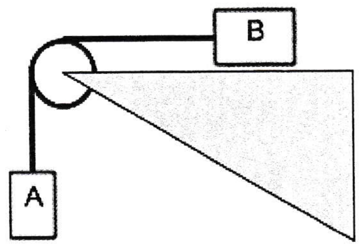
$F_{f_2} = \mu_2 (m_2 g + m_1 g) + \mu_1 m_1 g$

$m_2 a = F - \mu_2 (m_2 g + m_1 g) - \mu_1 m_1 g$

$m_2 \mu_1 g = F - \mu_2 (m_2 g + m_1 g) - \mu_1 m_1 g$

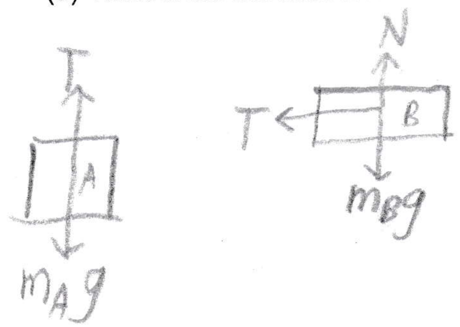
$$\boxed{F = \mu_1 m_2 g + \mu_2 m_2 g + \mu_2 m_1 g + \mu_1 m_1 g}$$

Problem 6



Blocks A and B have masses m_A and m_B, respectively. The friction is zero everywhere.

(a) What is the acceleration with which A is moving?



$$F = ma$$

$$m_A a = m_A g - T$$

$$T = m_B a$$

$$m_A a = m_A g - m_B a$$

$$m_A a + m_B a = m_A g$$

$$a = \boxed{\frac{m_A g}{m_A + m_B}}$$

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(b) What is the tension in the thread?

$$T = m_B a$$

$$T = m_B \left(\frac{m_A g}{m_A + m_B}\right)$$

$$T = \boxed{\frac{m_A m_B g}{m_A + m_B}}$$

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