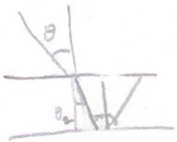
You must show your work to receive credit. An answer written down with no work will receive no credit.

(a): 4 pts

Consider a thin film of index of refraction $n > 1$ and thickness t, surrounded on either side by air ($n_{\text{air}} \approx 1$). Consider light of wavelength λ that is incident on this film at angle of incidence θ, where $0 \leq \theta < \pi/2$. Some of the light is refracted into the film, reflected off the bottom of the film, and then refracted back out into air. What is the distance travelled by the light while inside the film? [In terms of t, θ, λ, and/or n.]



$$n_{air} \sin(\theta) = n_{film} \sin(\theta_2)$$

$$\theta_2 = \sin^{-1}\left(\frac{n_{air}}{n_{film}} \sin(\theta)\right)$$

$$D = \frac{2t}{\cos(\theta_2)} = \frac{2t}{\cos(\sin^{-1}(\frac{n_{air}}{n_{film}} \sin(\theta)))} = \boxed{\frac{2t}{\cos(\sin^{-1}(\frac{\sin(\theta)}{n}))}}$$

(b): 8 pts

For the situation described in part (a), derive an expression for the wavelengths of light that will interfere *destructively* [In terms of t, θ, λ, and/or n. You may index these wavelengths by an integer m.]

$\Delta\phi_{tot} = \Delta_{dist} + \Delta\phi_{refl}$

$n > n_{air}$, so there will be π phase shift.

$$\frac{2t}{\cos(\sin^{-1}(\frac{\sin(\theta)}{n}))} = m\lambda$$

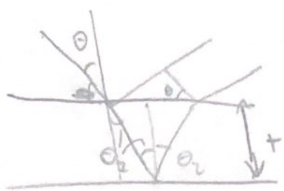
$$\boxed{\lambda = \frac{2t}{m\cos(\sin^{-1}(\frac{\sin(\theta)}{n}))}}$$

Problem 1 continued on next page...

You must show your work to receive credit. An answer written down with no work will receive no credit.

(c): 8 pts

When we derived the equations for thin-film interference, we neglected the fact that the wavelength of light changes when it changes medium. Modify your answer in part (b), taking this fact into account; i.e. derive an equation for the wavelengths (in air) of light that will interfere destructively when incident at angle θ on a thin film of thickness t and index of refraction n with air ($n_{\text{air}} \approx 1$) on either side. [In terms of t, θ, λ, and/or n. You may index these wavelengths by an integer m.]



Path difference $= x$

$$x = n\left(\frac{2t}{\cos\theta_2}\right) - 2t\tan(\theta_2)\, n \sin(\theta_2)$$

Path inside medium $= \dfrac{2t}{\cos(\theta_2)}$

$$\theta_2 = \sin^{-1}\left(\frac{\sin(\theta)}{n}\right)$$

$$x = 2nt\left(\frac{1-\sin^2(\theta_2)}{\cos(\theta_2)}\right)$$

$$x = 2nt\cos(\theta_2)$$

$$m\lambda = 2nt\cos\left(\frac{\sin(\theta)}{n}\right)$$

$$\lambda = \frac{2nt}{m}\cos\left(\frac{\sin(\theta)}{n}\right)$$

You must show your work to receive credit. An answer written down with no work will receive no credit.

(a): 15 pts

How far from the ~~second lens~~ mirror is the final image formed by the lens-mirror system? Make sure to specify whether the image is to the right or the left of the ~~lens~~. mirror

$$f_{lens} = \frac{1}{(1.6-1)\left(\frac{1}{15} - \frac{1}{10}\right)} = -50 \text{ cm}$$

$$f_{mirror} = \frac{-8}{2} = -4 \text{ cm}$$

$$s_1' = \frac{1}{-\frac{1}{50} - \frac{1}{80}} = -30.77 \text{ cm}$$

230.77 = distance from mirror

$$s_2' = \frac{1}{\left(-\frac{1}{4} - \frac{1}{230.77}\right)} = -3.93 = 3.93 \text{ cm}$$ to the right of the mirror

(b): 5 pts

What is the lateral magnification of the final image (with respect to the object)?

$$m_1 = -\frac{s'}{s} = \frac{-(-30.77)\text{cm}}{80 \text{ cm}} = 0.385$$

$$m_2 = -\frac{s'}{s} = \frac{-(-3.93 \text{ cm})}{230.77 \text{ cm}} = 0.017 \text{ cm}$$

$$m_{total} = m_1 \cdot m_2 = .00655$$

You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 3

20 pts

A long cylindrical conductor of radius R carries a uniform current density $\vec{J} = J\hat{z}$ that runs parallel to the axis of the cylinder (the z-axis). A time-varying electric field is established everywhere in space and is given by $\vec{E} = E_0 \frac{t}{t_0}\hat{z}$, where E_0, t_0 are constants of units V·m and s, respectively. Compute the magnetic field in the following regions: [You make take Maxwell's equations as given, but nothing else.] [answer in terms of $E_0, t_0, \mu_0, \epsilon_0, J$, and/or any spatial coordinates.]

(a): 10 pts

$r > R$

$$\oint B \cdot dl = \mu_0 \left(J_{encl} + \epsilon_0 \frac{d}{dt} \int_0^r E \cdot da \right)$$

$$B\, r \oint d\phi = \mu_0 \left(\pi r^2 J \hat{z} + \epsilon_0 \frac{E_0}{t_0} \hat{z} \cdot 2\pi r \right)$$

$$B\, 2\pi r = \mu_0 \left(\pi r^2 J \hat{z} + \epsilon_0 \frac{E_0}{t_0} \hat{z} \cdot 2\pi r \right)$$

$$B = \mu_0 \left(\frac{rJ}{2} + \epsilon_0 \frac{E_0}{t_0} \right)$$

Problem 3 continued on next page...

You must show your work to receive credit. An answer written down with no work will receive no credit.

(b): 10 pts

$r < R$

$$\oint B \cdot dl = u_0 \left(I_{encl} + \epsilon_0 \frac{d}{dt} \int E \cdot dA \right)$$

$$Br \oint d\theta = u_0 \left(J \frac{\pi r^2}{\pi R^2} + \epsilon_0 \frac{E_0}{t_0} 2\pi r \right)$$

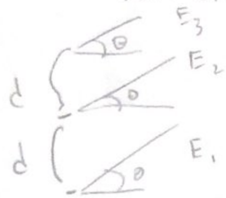
$$2\pi r B = u_0 \left(J \frac{\pi r^2}{\pi R^2} + \epsilon_0 \frac{E_0}{t_0} 2\pi r \right)$$

$$B = u_0 \left(\frac{J r}{2\pi R^2} + \epsilon_0 \frac{E_0}{t_0} 2\pi r \right)$$

You must show your work to receive credit. An answer written down with no work will receive no credit.

Path difference x

$$E = E_1 + E_2 + E_3$$



$$E = E_0 \sin(\omega t) + E_0 \sin(\omega t + \phi) + E_0 \sin(\omega t + 2\phi)$$

$$\phi = \frac{2\pi}{\lambda} x = \frac{2\pi}{\lambda} d \sin\theta$$

$$\boxed{E = E_0 \left(\sin(\omega t) + \sin(\omega t + \frac{2\pi}{\lambda} d \sin\theta) + \sin(\omega t + \frac{4\pi}{\lambda} d \sin\theta) \right)}$$

You must show your work to receive credit. An answer written down with no work will receive no credit.

Problem 5

20 points

(a): 5 pts

Suppose you throw a particle of charge q at velocity $\vec{v} = v\hat{z}$ along the axis of a straight, infinitely long solenoid of radius R carrying constant current I. Will the particle feel a force? If so, calculate the force (magnitude and direction).

The particle will not feel a force, Because the current is constant, there is no induced electric field. Additionally, $F = qvB\sin\theta$, and because the magnetic field of a solenoid points along its axis, the velocity and magnetic field vectors are parallel, Thus $\sin\theta = 0$, so $F = 0$.

(b): 5 pts

Suppose that I place a device at a particular point in space that measures the electric field at that point. The device finds that the electric field at that location is oscillating sinusoidally in the $\hat{z}$ direction with amplitude E_0 and angular frequency ω: $\vec{E} = E_0 \cos(\omega t)\hat{z}$. Is this information alone sufficient to determine the *intensity* of the corresponding electromagnetic wave? Explain your answer.

Yes, the intensity can be found through $I = \frac{1}{2} c \epsilon_0 E_0^2$.

Problem 5 continued on next page...

You must show your work to receive credit. An answer written down with no work will receive no credit.

(c): 5 pts

Suppose that I have a beam of light linearly polarized in the $+\hat{z}$ direction, propagating in the $+\hat{y}$ direction. I would like to obtain light linearly polarized in a direction tilted an angle ϕ from the z-axis. I could do so in several ways, two of which are:

1. Send the light through a single polarizing filter whose polarizing axis is tilted at an angle ϕ from the z-axis.
2. Send the light through *two* polarizing filters, the first of which has its axis an angle $\phi/2$ from the z-axis, and the second of which has its axis an angle ϕ from the z-axis.

If I am interested in maximizing the intensity of the light passing through the filters, which of these options should I choose? Or do they both yield the same intensity? Justify your answer.

You should choose the 2nd option. Intensity of light through a polarizing filter can be given by $I = I_0 \cos^2(\Theta)$, where theta is the angle between the direction of polarization and the filter's axis. Consider the case where ϕ is $90°$. Using a single filter will cause no light to pass through, because $I_0 \cos^2(90) = 0$. However, 2 filters with angle $\phi = 45°$ will allow $\frac{1}{4}$ of the original intensity to pass through. The same concept can be applied to other angles of ϕ, where $(I_0 \cos(\frac{\phi}{2}))\cos(\frac{\phi}{2})$ is greater than $I_0 \cos(\phi)$.

You must show your work to receive credit. An answer written down with no work will receive no credit.

(d): 5 pts

Consider the circuit shown. The circuit consists of an AC voltage source with amplitude V_0 and angular frequency ω, an inductor L, a capacitor C, and two arbitrary machines A and B that behave like resistors. If I want to increase the current running through machine A, which of the following should I do?

1. Increase the driving frequency.
2. Decrease the driving frequency.
3. There is not enough information to determine.

Choose one and justify your answer.

There is not information to determine. The initial driving frequency must be assessed. If the initial frequency is below the resonance frequency $\omega = \frac{1}{\sqrt{LC}}$, the driving frequency must be increased, and vice versa for if the initial frequency is greater than the resonance frequency.