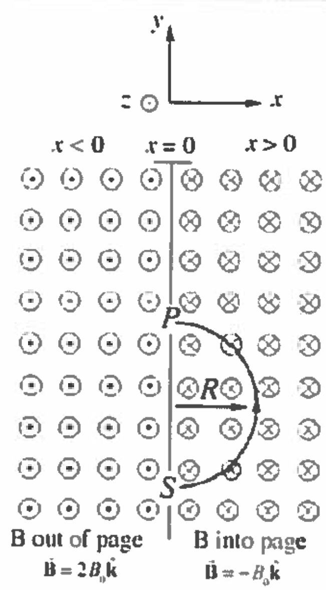
Physics 1C – Spring 2018: Midterm 1

Name Tianyang Peng Discussion Time

This exam is closed book and closed notes. Electronics are not permitted, except for one calculator. Please show your full solution in the boxes provided (where the scanners can pick them up). Your solutions will be graded on correctness and coherence; results given with no details will receive zero credit. There is additional scratch paper attached so you can collect your thoughts first. Academic dishonesty is reported to the Office of the Dean of Students.

Problem 1. The x-y plane for $x < 0$ is filled with a uniform magnetic field pointing out of the page, $\mathbf{B} = 2B_0\hat{k}$ with $B_0 > 0$, as shown. The x-y plane for $x > 0$ is filled with a uniform magnetic field $\mathbf{B} = -B_0\hat{k}$, pointing into the page, as shown. A charged particle with mass m and charge q is initially at the point S at x=0, moving in the positive x- direction with speed v. It subsequently moves counterclockwise in a circle of radius R, returning to x = 0 at point P, a distance 2R from its initial position, as shown in the sketch.



a. Is the charge positive or negative? Briefly explain your reasoning.

It's positive. Using the right-hand rule, the force is acting on +y direction. The charge moves accordingly to the centripetal force.

b. Find an expression for the radius R of the trajectory shown, in terms of v, m, q and B_0 as needed.

$$\frac{mv^2}{R} = q\vec{v}\times\vec{B} = q|v||B| \quad \swarrow \quad \vec{v}\perp\vec{B}$$

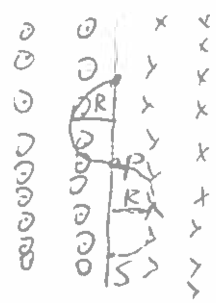
$$R = \frac{mv}{qB} = \frac{mv}{q|2B_0|}$$

c. How long does the particle take to return to the plane x=0 at point P, in terms of v, m, q and B_0 as needed?

Distance : $\frac{2\pi R}{2} = \pi R$

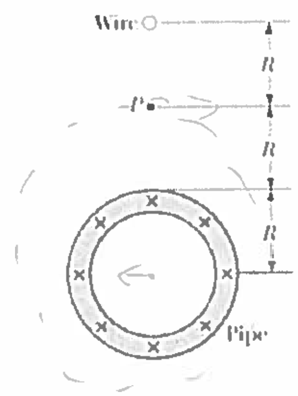
$$T = \frac{2\pi R}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB} = \frac{\pi m}{qB_0} \ s$$

d. Describe and sketch the entire subsequent trajectory of the particle after it passes point P. Define any relevant distances in terms of v, m, q and B_0.



It will move clockwise after point P under a circular motion of radius R. After t=T from the last problem, it will return to x=0 and start a circular motion the same as previous part of the problem. Its motion will flip back and forth periodically.

Problem 2. In the figure below a long circular pipe with outside radius R carries a (uniformly distributed) current I into the page. A long wire runs parallel to the pipe at a distance of 3.00R from center to center. Find the current in the wire such that the ratio of the magnitude of the net magnetic field at point P to the magnitude of the net magnetic field at the center of the pipe is x, but it has the opposite direction.



$B_p = \frac{\mu_0 I_w}{2\pi R}$ due to wire (I_w = current of wire)

$\oint B_p \cdot dl = \mu_0 I$

$B_p = \frac{\mu_0 I}{(2R)2\pi} = \frac{\mu_0 I}{4\pi R}$

$B_p = \frac{\mu_0 I_w}{2\pi R} - \frac{\mu_0 I}{4\pi R} = \frac{2\mu_0 I_w - \mu_0 I}{4\pi R}$

$B_c = \frac{\mu_0 I_w}{2\pi 3R} = \frac{\mu_0 I_w}{6\pi R}$ only due to wire.

$\frac{2\cancel{\mu_0} I_w - \cancel{\mu_0} I}{\cancel{4\pi R}\ 2} \cdot \frac{\overset{3}{\cancel{6\pi R}}}{\cancel{\mu_0} I_w} = x$

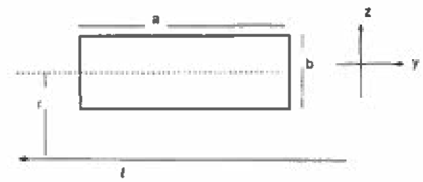
$\frac{6I_w - 3I}{2I_w} = x$

$(6-2x)I_w = 3I$

$I_w = \frac{3}{6-2x} I$, I is the current in the pipe.

I_w is into the page.

Problem 3. A rectangular loop of wire with length a, width b, and resistance R is placed near an infinitely long wire carrying current i, as shown in the figure . The distance from the long wire to the center of the loop is r.



$\otimes B$

a. Find an expression for the total flux through the loop.

$$\Phi = \int B \cdot dA$$

$$B = \frac{\mu_0 I}{2\pi z}$$

$$\Phi = \int_0^a \int_{r-\frac{b}{2}}^{r+\frac{b}{2}} \frac{\mu_0 I}{2\pi z}\, dz\, dy = \int_0^a \frac{\mu_0 I}{2\pi} \ln\left|\frac{r+\frac{b}{2}}{r-\frac{b}{2}}\right| dy = \frac{\mu_0 I a}{2\pi} \ln\left|\frac{r+\frac{b}{2}}{r-\frac{b}{2}}\right|$$

b. What is the magnitude and direction of the current flowing in the circuit as it is pulled away from the wire with velocity $\mathbf{v} = v_0\hat{k}$.

Direction : Clockwise

$$B = \frac{\mu_0 I}{2\pi z}$$

$$= \iint \frac{dB}{dt} dA$$

$$= -\int_0^a \int_{r-\frac{b}{2}}^{r+\frac{b}{2}} \frac{\mu_0 I V_0}{2\pi z^2}\, dz\, dy$$

$$+ \int_0^a \frac{\mu_0 I V_0}{2\pi}$$

$$\Phi = \frac{\mu_0 I a}{2\pi} \ln\left|\frac{r+\frac{b}{2}}{r-\frac{b}{2}}\right|$$

$$r = r_0 + V_0 t$$

$$\Phi(t) = \frac{\mu_0 I a}{2\pi} \ln\left|\frac{r+V_0t+\frac{b}{2}}{r+V_0t-\frac{b}{2}}\right| \qquad \frac{d\Phi}{dt} = \frac{\mu_0 I a}{2\pi}\left(\frac{r+V_0t-\frac{b}{2}}{r+V_0t+\frac{b}{2}}\right)$$

$$= \frac{\mu_0 I a}{2\pi}\left(\ln\left|r+V_0t+\frac{b}{2}\right| - \ln\left|r+V_0t-\frac{b}{2}\right|\right)$$

$$\frac{d\Phi}{dt} = \frac{\mu_0 I a}{2\pi}\left(\frac{V_0}{r+V_0t+\frac{b}{2}} - \frac{V_0}{r+V_0t-\frac{b}{2}}\right)$$

$$I = \frac{-\frac{d\Phi}{dt}}{R} = \frac{\mu_0 I a}{2\pi R}\left(\frac{V_0}{r+V_0t-\frac{b}{2}} - \frac{V_0}{r+V_0t+\frac{b}{2}}\right)$$

c. Show that to maintain this motion, the rate at which the external force is doing work on the loop is equal to the rate at which energy is being dissipated in the loop.

$$\vec{F} = I \times a \times B = I a \left(\frac{\mu_0 i}{2\pi(r + v_0 t + \frac{b}{2})} - \frac{\mu_0 i}{2\pi(r + v_0 t - \frac{b}{2})} \right)$$

$$F \cdot v_0 = I a v_0 \left(\frac{\mu_0 i}{2\pi(r + v_0 t + \frac{b}{2})} - \frac{\mu_0 i}{2\pi(r + v_0 t - \frac{b}{2})} \right)$$

$$= \frac{(\mu_0 a v_0 i)^2}{(2\pi R)^2 R} \left(\frac{1}{r + v_0 t + \frac{b}{2}} - \frac{1}{r + v_0 t - \frac{b}{2}} \right)^2$$

P in the current:

$$P = I^2 R = \frac{(\mu_0 i a v_0)^2}{(2\pi R)^2 R} \left(\frac{1}{(r + v_0 t + \frac{b}{2})} - \frac{1}{(r + v_0 t - \frac{b}{2})} \right)^2$$

$$P = F \cdot v_0$$

Scratch paper