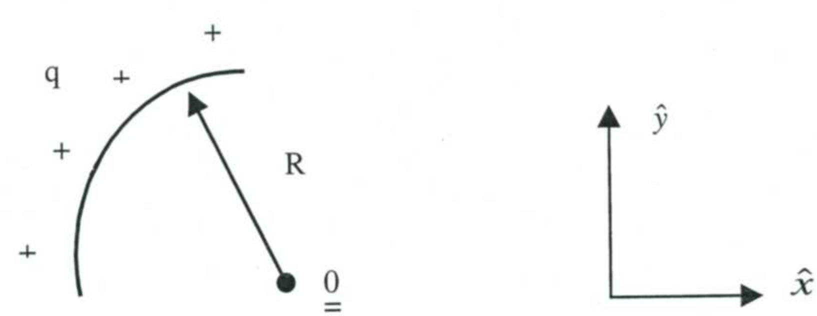
Physics 110A Spring 2004

Final Exam **Prof. G.J. Morales**

Problem #1 – 10 points

Consider a uniform line charge carrying a total amount of charge q and consisting of one quarter of a circle of radius R, as shown below



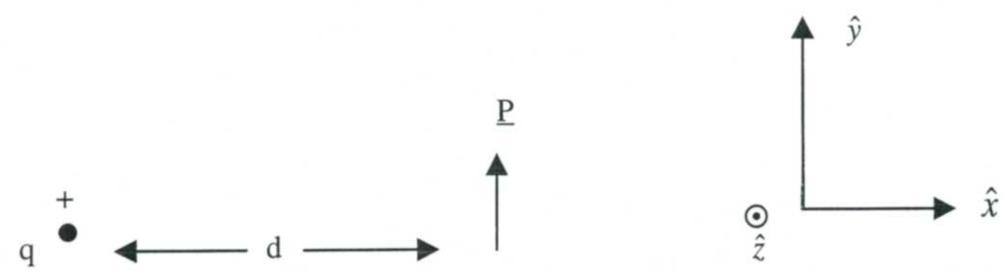
a) What is the line charge density λ of this configuration? (2 points)

b) Calculate the electrostatic potential ϕ at the origin $\underline{0}$ (the center of the circle). (3 points)

c) Calculate the electric field vector $\underline{E}$ at $\underline{0}$. Express answer answer in terms of the unit vectors $\hat{x}, \hat{y}$. (5 points)

Problem #2 – 12 points

An electric dipole $\underline{p} = p\,\hat{y}$ is located at a distance d from a positive point charge q, as shown below



The direction of $\underline{p}$ is a right-angles to the line connecting the charge.

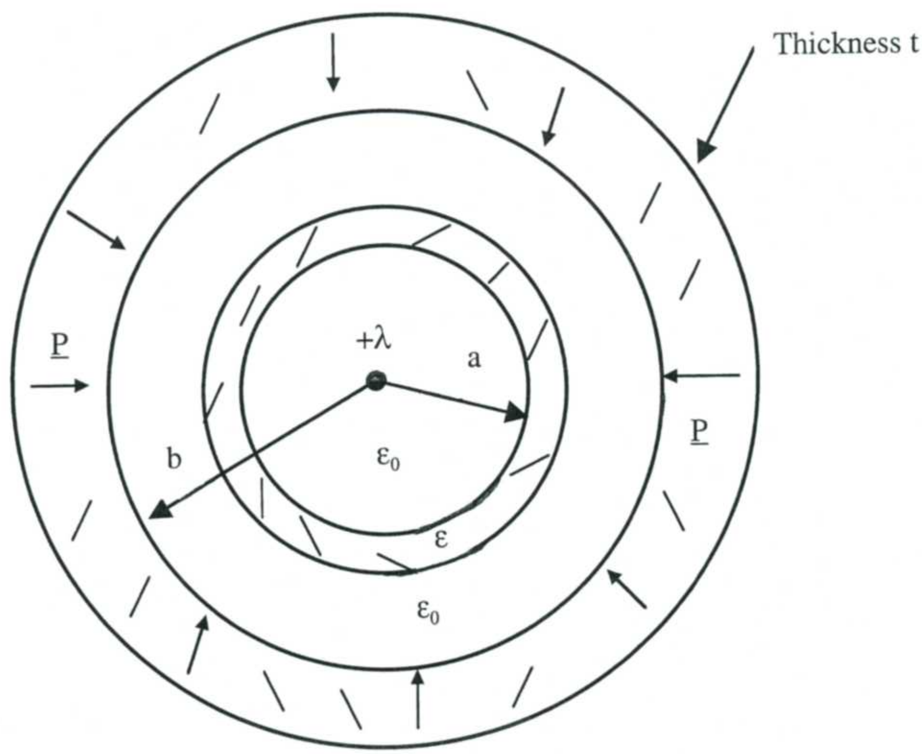
a) Find the torque vector $\underline{\tau}$ experienced by the dipole. (4 points)

b) Find the force vector $\underline{F}$ acting on the dipole. (5 points)

c) How much work W is required to bring the $+q$ charge from ∞ to its location in the figure. (3 points)

Problem #4 – 20 points

An infinitely long line of free charge having line density λ (Coulomb/meter) is located at $r = 0$. It is surrounded by two infinitely long concentric cylindrical shells, both having thickness t. The inner shell is a normal dielectric material having dielectric coefficient ϵ and whose inner surface is at a distance $r = a$ from λ. The outer shell consists of an electret material whose polarization vector is $\underline{P} = -P_0\hat{r}$, with P_0 a positive constant. The inner surface of the electret is located at $r = b$ from λ, as indicated below



a) Find the displacement vector $\underline{D}$ at an arbitrary radius r. (5 points)

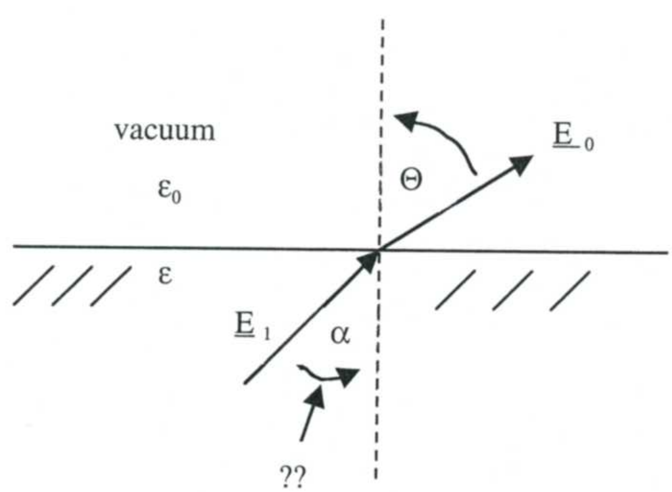
b) Find the bound surface charge density σ_b on the outer surface of the dielectric shell (i.e., at $r = a + t$). (5 points)

c) Find the bound charge density ρ_b at a point r inside the electret. (5 points)

d) Find the electric field $\underline{E}$ at a point r inside the electret. (5 points)

Problem #3 – 10 points

It is found that in the absence of free charges the electric field $\underline{E}_0$ on the vacuum side of an interface with a material having dielectric coefficient ϵ makes an angle θ with the local normal vector, as shown below



a) Find the angle α that the electric field vector $\underline{E}_1$ on the dielectric side makes with the local normal. (5 points)

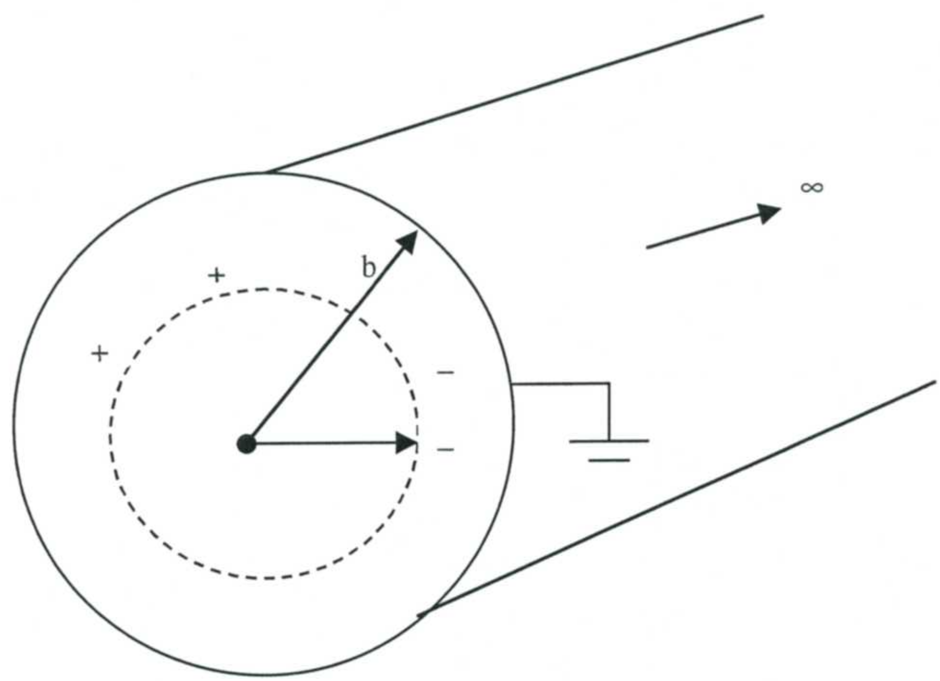
b) Find the ratio of the electric field energy density on the material side to that on the vacuum side. (5 points)

Problem #5 – 13 points

The potential on the surface of an infinitely long cylinder of radius a is prescribed to have an azimuthal variation given by

$$\phi(r = a, \theta) = \phi_0 \cos 3\,\theta$$

A second infinitely long cylinder of radius b, and concentric with the inner one, is grounded, as indicated below

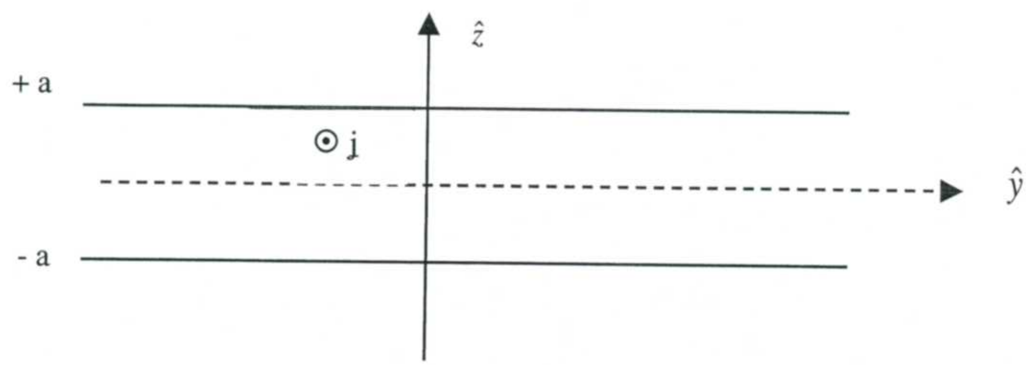


a) Find the potential $\phi(r, \theta)$ at a radius r between the two cylinders, i.e., $a < r < b$. (8 points)

b) Find the surface charge induced on the grounded cylinder. (5 points)

Problem #7 – 15 points

A slab (current sheet) extending from $-a$ to $+a$ in the z-direction and infinite along the x and y–directions carries a uniform current density $\underline{j} = j\hat{x}$, as shown below



Find the magnetic field vector $\underline{B}$ for a point z inside and outside the slab.

Problem #8 – 10 points

An electric charge distribution produces an electric field

$$\underline{E} = c(1 - e^{-\gamma r})\frac{\hat{r}}{r^2}$$

where c and γ are constants and r is the radical spherical coordinate.

Find the net charge within the radius $r = 1/\gamma$.