105A -Take Home Midterm Exam 2 (COVID Measures)

This is an open book take home exam. You can use whatever materiel you have, just work alone. It is due on **Friday, Feb 25 at 8 am** - no late submission[1] . I suggest you'll organize your time accordingly so you'll have enough time to check your work.

Honor system declaration (sign below)

A work with no signature will not be accepted

I ( ) state that I have worked on this take home exam by myself and did not discussed (asked/talked/texted/chatted/tweeted/... etc) with **anyone** on its content/problems/solutions.

Signature:

General grading rubric: partial credit will be given so show your work.
No dragging algebraic error.
Unit error = -20% of the entire sub section, even if it's only on one part, this will continue for any sub section, so check your units!

Questions:

(Grades are out of 150)

Two drank† ants are having a trust exercise. They are tied together with a massless string of length a. The ant with a mass m_1 is set on the surface of a can shaped like a cylinder of radius R, while the second ant with a mass m_2 is forced to move on the inner side of the cylinder. A small hale was drilled at the center of the cylinder and the string is passed through it (see figure for an illustration of the system - the left cartoon simplify the problem if you are confused by the ants). The cylinder height is much longer than the length of the string.
Gravity points down.

†*Of course, these are the same ants from our calculus of variations lecture and this is why they are so drank.*

[1] CAE students have their own predetermined schdule.

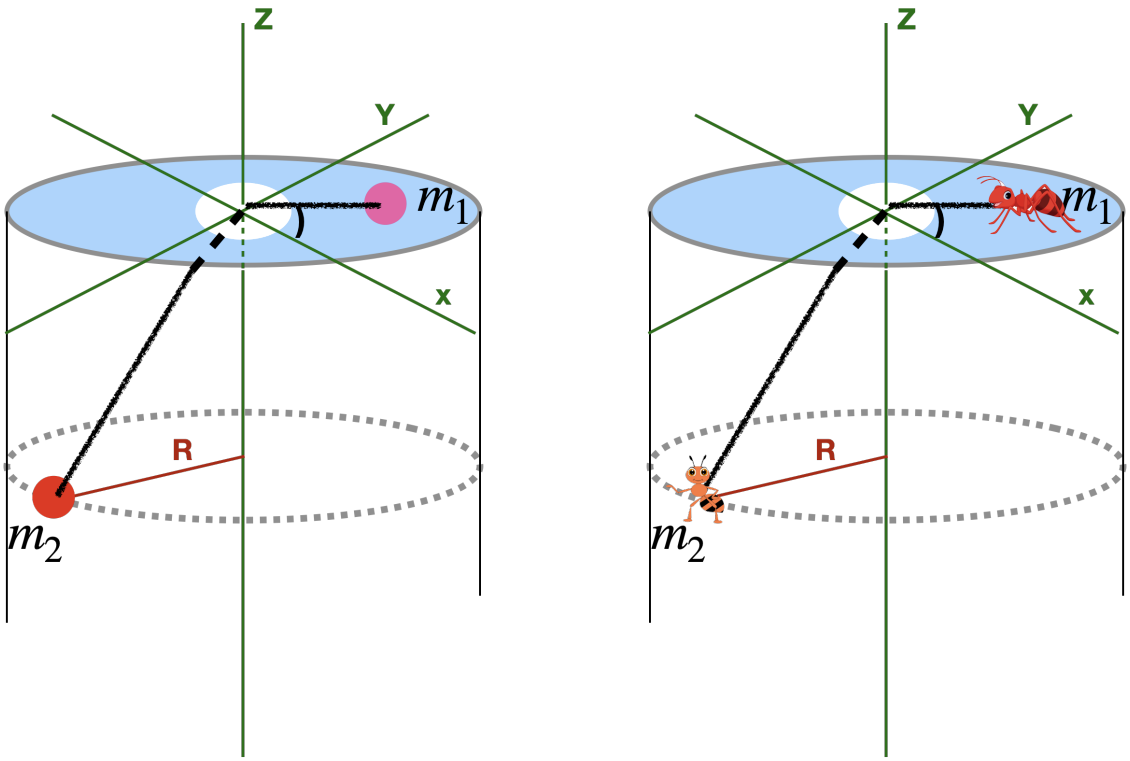

Figure 1: The system's configuration. To the left I show the system representing the two ants as two masses (in case you're finding the setting confusing).

1. (6pt) How many degrees of freedom the system has?
2. (24pt) Choose generalized coordinates and express the transformation from cartesian coordinates to the generalized coordinates you choose.
3. (25pt) Find the Lagrangian of the system.
4. (18pt) Find the conjugate momenta.
5. (24pt) Find the integral of motions if the system (=18pt). Explain what are the symmetries associated with these integral of motions (=6pt).
6. (23pt) Write the equation of motion(s).
7. (15pt) A student thought to approach the problem as follow: they considered the center of mass of the two masses (the two ants) using (x_c, y_c, z_c), for which

$$(m_1 + m_2)x_c = m_1 x_1 + m_2 x_2 \quad (1)$$
$$(m_1 + m_2)y_c = m_1 y_1 + m_2 y_2 \quad (2)$$
$$(m_1 + m_2)z_c = m_1 z_1 + m_2 z_2 \quad (3)$$

And then define a separation between the two masses to be:

$$x = x_2 - x_1 \tag{4}$$
$$y = y_2 - y_1 \tag{5}$$
$$z = z_2 - z_1 \tag{6}$$

The student thought that because the two masses are tie together, it make sense to choose these two coordinates because the string length doesn't change and it is always a. Does the student's reasoning make sense? Are these choices beneficial?

8. (15pt) The student then concluded that the momentum associated with the center of mass vector is conserved, and that they can move the frame of reference to the center of mass.
   Is the student correct in their conclusion? Explain your answer.