105A -Take Home Midterm Exam (COVID Measures)

This is an open book take home exam. You can use whatever materiel you have, just work alone. It is due on **Thursday, Feb 10 at 8 am** - no late submission[1] . I suggest you'll organize your time accordingly so you'll have enough time to check your work.

Honor system declaration (sign below)

A work with no signature will not be accepted

I (   ) state that I have worked on this take home exam by myself and did not discussed (asked/talked/texted/chatted/tweeted/... etc) with **anyone** on its content/problems/solutions.

Signature:

General grading rubric: partial credit will be given so show your work.
No dragging algebraic error.
Unit error = -20% of the entire sub section, even if it's only on one part, this will continue for any sub section, so check your units!

Questions:

(Grades are out of 150)

A rubber ducky with mass M floats near the edge of an inflatable pool. The drag by the water on the ducky depends on the speed v of the ducky as $-bv$, where $b =$ Const. In a windy day you find that ducky is bobbing back and front with a restoring force that scales as $-5b^2x/(2M)$. On that day a kid attached a massless string to the ducky and is ready to pull the it along the pool.

1. (15pt) Just before the kid pulls the ducky, the ducky is at rest. A dragonfly with mass $m << M$ flies with a velocity v_0 hits the ducky and stick to it. Find the ducky's initial velocity. Which conservation law are you using to solve this?
2. (28pt) The kid begins to pull the ducky with a constant force $f_0 = 5bv_D$, where v_D is the ducky's velocity right after the collision with the dragonfly. Write the equation of motion.
   Hint: Do not use Lagrange formalism.

[1] CAE students have their own predetermined schdule.

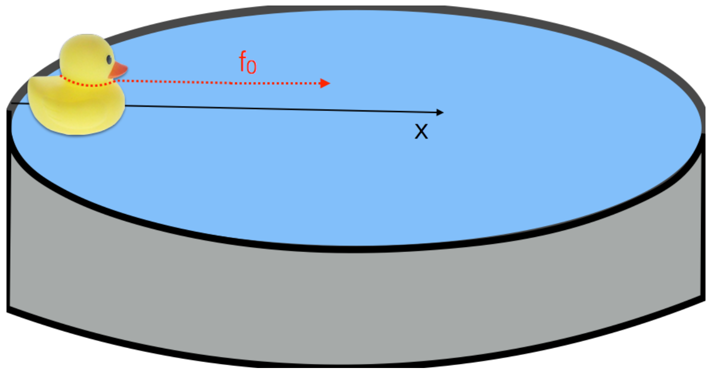

Figure 1: The system's configuration

3. (30pt) Find the frequency of oscillations in terms of b and M.

4. (36pt) Find the expression for the distance of the ducky as a function of time $x(t)$. In other words, solve the equation of motion
   Express your answer as, only, a function of b, M, v_0 and m,
   Hints:
   (1) Remember to use the initial condition with the dragonfly.
   (2) First find the homogenous solution and remember to add the particular solution.

5. (18pt) The kid is now pushing the ducky back and forth with some frequency. A student during the exam wanted to have a discussion about which frequency will be the easiest for the kid to push the ducky back and forth. However, because the student is taking the exam and signing the honor declaration, they had this discussion with their imaginary friend. The imaginary friend thought that it will be the easiest to push the ducky back and forth with the oscillatory frequency (the one you found in 3) on the same phase, while the student thought that the kid needs to push the ducky back and forth with a frequency that will be much larger than the damping over the mass (i.e., b/M), to over come the damping force.

   Who is correct (if anyone), explain your answer.

   (It was a horrible argument and now the student and their imaginary friend are not talking with each other.)

6. (12pt) During a different day, when the kid is outside with the ducky you find that the wind's speed is slower and the ducky is is bobbing back and front with a restoring force that scales as $-b^2x/(4M)$ (instead of the restoring force you had before!). Describe the type of motion the ducky will have in the absence of the kids pull. Explain your answer.

7. (11pt) In which case (the bobbing with restoring force of $-b^2x/(4M)$ vs $-5b^2x/(2M)$) will the ducky reach further (meaning the displacement from $t = 0$ to $t \to \infty$), assuming the same initial condition? Explain your answer.
   It may helpful to you to have the following in initial conditions (but not entirely necessary): $x(t = 0) = x_0$ and $v(t = 0) = \frac{-bx_0}{2M}$.