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Physics 105A
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P. Kraus

Midterm Exam

Problem 1: 15

Problem 2: ~~16~~ 18 PK

Problem 3: 19

Total: ~~50~~ /60
52 PK

To get credit for an answer you must show your work!

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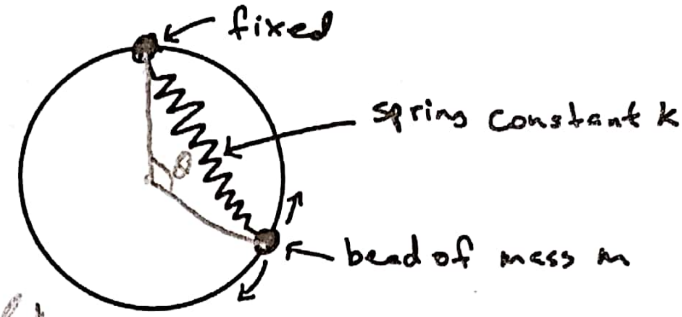
Problem 1: [20 points]

A bead is confined to slide on a circular hoop of radius R, as shown below. **There is no gravity in this problem**; however the particle is attached to a spring whose other endpoint is fixed at the top of the spring. The spring itself is not constrained to lie on the hoop.

a) Write the action for this system, specifying your generalized coordinate(s)

b) Write the Euler-Lagrange equations

c) How many static (bead is stationary) solutions are there, and indicate whether they are stable or unstable to small perturbations. You don't need to do any computations here, just give a short explanation in words.



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a) coordinate: θ in radians around hoop
origin = fixed point, $\theta=0$ at fixed point

action = ?

$x_{bead} = R\sin\theta$
$y_{bead} = R\cos\theta - R$

d = distance from origin $=\sqrt{x^2+y^2} = \sqrt{R^2\sin^2\theta + R^2\cos^2\theta - 2R^2\cos\theta + R^2}$
$= \sqrt{2R^2 - 2R^2\cos\theta}$

b) $T = \frac{1}{2}mv^2 = \frac{1}{2}mR^2\dot{\theta}^2$

$U = \frac{1}{2}kd^2 = \frac{1}{2}k(2R^2 - 2R^2\cos\theta) = kR^2(1-\cos\theta)$

$L = T - U = \frac{1}{2}mR^2\dot{\theta}^2 - kR^2(1-\cos\theta)$

$$\frac{\partial L}{\partial \theta} = \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} \longrightarrow -kR^2\sin\theta = \frac{d}{dt}(mR^2\dot{\theta}) = mR^2\ddot{\theta} \longrightarrow \boxed{\ddot{\theta} = -\frac{k}{m}\sin\theta}$$ ✓ 7/7

7/7

c) Any time that the force on the bead (from the spring) is <u>not</u> perpendicular to the bead's hoop constraint, the bead has an unbalanced force and is thus not at equilibrium. There are only 2 points where this is not the case:

$\theta = \pi$ → unstable, since small perturbations will cause an unbalanced force pointing away from the point ✓

$\theta = 0$ → stable, since small perturbations will cause an unbalanced force pointing back to the point ✓

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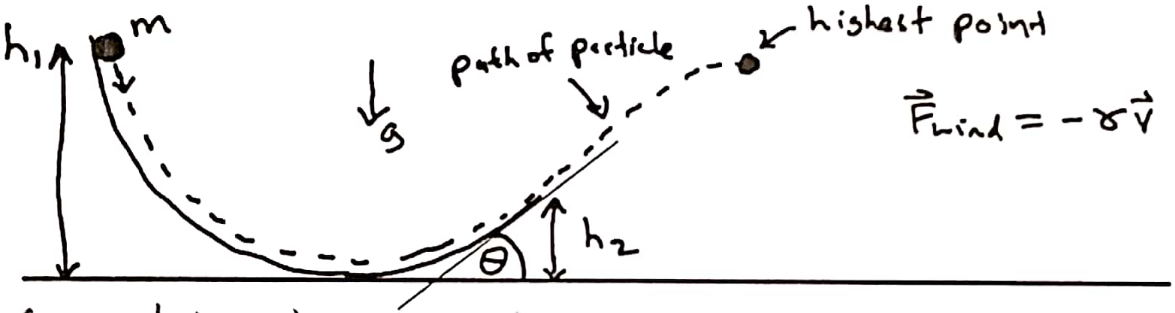
Problem 2: [20 points]

A particle of mass m is released from rest on a frictionless track, as shown. When it comes to the end of the track the particle flies into the air at angle θ with the horizontal, where it encounter wind resistance of the form $\vec{F} = -\gamma\vec{v}$. There is no wind resistance while the particle is on the track.

a) How much time elapses between the particle leaving the track and it reaching its highest point?

b) What is the height of the particle at its highest point?

c) How much horizontal distance does the particle travel between leaving the track and when it reaches its highest point?



a) v_0 = speed of particle as it leaves track

wind resistance is independent between coordinates, since it's linear

$v_{oy} = v_0 \sin\theta$

$$a_y = \frac{\Sigma F_y}{m} = \frac{-mg - \gamma v_y}{m} = -g - \frac{\gamma}{m}v_y = \frac{dv_y}{dt}$$

$$\int_{v_{oy}}^{0} \frac{dv_y}{-g-\frac{\gamma}{m}v_y} = \int_0^t dt = t \longrightarrow t = \int_0^{v_{oy}} \frac{dv_y}{g+\frac{\gamma}{m}v_y} = \frac{m}{\gamma}\ln\left(g+\frac{\gamma}{m}v_y\right)\Big]_0^{v_{oy}} = \frac{m}{\gamma}\ln\left(\frac{g+\frac{\gamma}{m}v_0\sin\theta}{g}\right)$$

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$$\boxed{t_h = \frac{m}{\gamma}\ln\left(1+\frac{\gamma}{mg}v_0\sin\theta\right)} \longleftarrow \text{call this quantity } t_h$$

b) $$\int_{v_{oy}}^{v_y} \frac{dv}{-g-\frac{\gamma}{m}v_y} = \int_0^t dt = t \to t = \int_{v_y}^{v_{oy}} \frac{dv_y}{g+\frac{\gamma}{m}v_y} = \frac{m}{\gamma}\ln\left(g+\frac{\gamma}{m}v_y\right)\Big]_{v_y}^{v_{oy}} = \frac{m}{\gamma}\ln\left(\frac{g+\frac{\gamma}{m}v_0\sin\theta}{g+\frac{\gamma}{m}v_y}\right) = t$$

$$\frac{g+\frac{\gamma}{m}v_0\sin\theta}{g+\frac{\gamma}{m}v_y} = e^{\frac{\gamma}{m}t} \longrightarrow g+\frac{\gamma}{m}v_y = \left(g+\frac{\gamma}{m}v_0\sin\theta\right)e^{-\frac{\gamma}{m}t}$$

$$v_y = \frac{m}{\gamma}\left(g+\frac{\gamma}{m}v_0\sin\theta\right)e^{-\frac{\gamma}{m}t} - \frac{mg}{\gamma}$$

$$y_h = \int_0^{t_h} v_y\,dt = -\frac{m^2}{\gamma^2}\left(g+\frac{\gamma}{m}v_0\sin\theta\right)e^{-\frac{\gamma}{m}t} - \frac{mg}{\gamma}t\Big]_0^{t_h} + h_2$$

plug in t_h

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$$\boxed{y_h = h_2 + \frac{m^2}{\gamma^2}\left(g+\frac{\gamma}{m}v_0\sin\theta\right) - \frac{m^2}{\gamma^2}\left(g+\frac{\gamma}{m}v_0\sin\theta\right)e^{-\frac{\gamma}{m}t_h} - \frac{mg}{\gamma}t_h}$$

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c) $a_x = \frac{\Sigma F_x}{m} = \frac{-\gamma v_x}{m} = \frac{dv_x}{dt}$ $\quad v_{0x} = v_0 \cos\theta$

$$\int_{v_{0x}}^{v_x} \frac{dv_x}{v_x} = \int_0^t -\frac{\gamma}{m}\,dt = -\frac{\gamma}{m}t$$

$$\ln\left(\frac{v_x}{v_{0x}}\right) = -\frac{\gamma}{m}t$$

$$v_x = v_{0x}e^{-\frac{\gamma}{m}t} = v_0\cos\theta e^{-\frac{\gamma}{m}t}$$

$$x_h = x(t_h) = \int_0^{t_h} v_x\,dt = -\frac{m}{\gamma}v_0\cos\theta e^{-\frac{\gamma}{m}t}\Big]_0^{t_h} = \frac{m}{\gamma}v_0\cos\theta\left(1 - e^{-\frac{\gamma}{m}t_h}\right)$$

$$\boxed{x_h = \frac{m}{\gamma}v_0\cos\theta\left(1 - e^{-\frac{\gamma}{m}t_h}\right)}$$ t_h from previous page

~~plug in t_h 4/6~~

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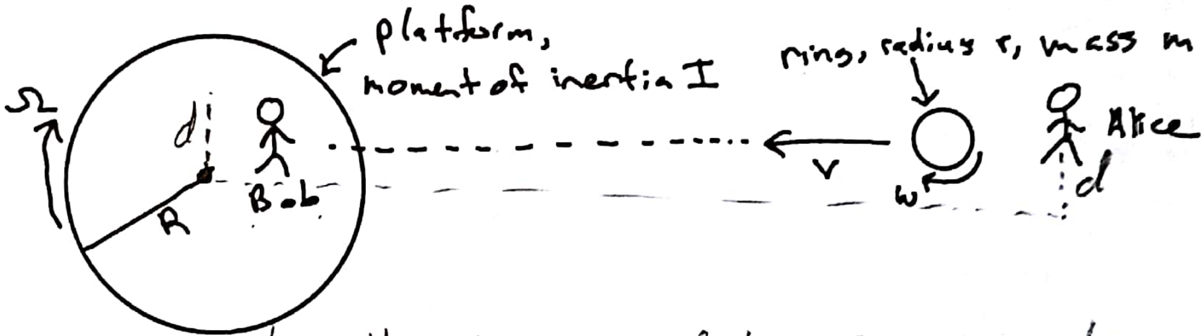
Problem 3: [20 points] **Name**

Bob is standing on a round platform of radius R and moment of inertia I that can rotate without friction. Alice is standing away from the platform. She can throw frisbees to Bob, who will catch them. The frisbees are taken to be thin rings of radius r and mass m. The frisbees spin at angular velocity ω, in the clockwise direction as viewed from above. Alice can throw the frisbee so that its center of mass travels at speed v. Alice's goal is to throw frisbees to Bob and make the platform rotate at angular velocity Ω, in the clockwise direction as viewed from above. You can assume that Bob drops the frisbees off the platform after catching them, so the mass of platform does not change.and that Bob has negligible mass.

a) Where should Alice and Bob stand to accomplish the goal with as few frisbee throws as possible?

b) What is the change in angular momentum of the platform with each catch?

c) How many frisbee throws does it take to accomplish the goal?



b) The only location that matters here is the distance from the center perpendicular to the frisbee's path. We'll call this $\underline{d}$

$$I_{frisbee} = \int_0^r r^2 dm = \int_0^r r^2 (2\pi r\,dr) = \frac{\pi}{2} r^4 \Big]_0^r = \frac{\pi}{2} r^4$$

$$L_{frisbee} = L_{rel} + L_{CoM} = I_{fris}\omega + mvd$$

$\underline{L_{platform}}$ increases by $\underline{L_{frisbee}}$ with each throw → $\boxed{L_{fris} = I_{fris}\omega + mvd}$

a) Bob should stand right at the edge of the platform, & Alice should stand where her throw will follow tangential path to Bob & the platform

- this increases the angular momentum of each throw

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c) $L_{platform} = I\Omega = \underbrace{n}_{\#\text{ of throws}}(I_{fris}\omega + mvd)$

$$n = \frac{I\Omega}{I_{fris}\omega + mvd}$$