Name: ________________
Student ID Number: ________________
Discussion Section: ________________

Physics 105A
Midterm Exam

You have one hour and fifteen minutes to complete this exam. You are allowed to use the one page of notes. You will not need a calculator (but you may have one if you like). There are four questions. Each is worth twenty-five points. There will be partial credit given in each of them. The point value of each part of each question is shown next to that part. Not all questions are equally difficult. Please do the easier ones first and do as much of each one as you can.

Please put your name on each page that you turn in. Good luck!

Question	Score
1	
2	
3	
4	
Total	/100

Some potentially useful formulae

- $\int \frac{dx}{C-x^2} = \frac{1}{\sqrt{C}} \tanh^{-1}\left(\frac{x}{\sqrt{C}}\right)$
- $\int \frac{dx}{C+x^2} = \frac{1}{\sqrt{C}} \tan^{-1}\left(\frac{x}{\sqrt{C}}\right)$
- $\int dx \cos^2(x) = \frac{x}{2} + \frac{1}{4}\sin(2x)$
- $\cosh^2(x) - \sinh^2(x) = 1$
- $\cosh(x \pm y) = \cosh(x)\cosh(y) \pm \sinh(x)\sinh(y)$
- $\sinh(x \pm y) = \sinh(x)\cosh(y) \pm \cosh(x)\sinh(y)$
- $\tanh^{-1}(x) = \frac{1}{2}\ln\left[\frac{1+x}{1-x}\right]$
- $\tanh\left[\cosh^{-1}(x)\right] = \frac{\sqrt{x^2-1}}{x}$
- $\tanh\left[\sinh^{-1}(x)\right] = \frac{x}{\sqrt{x^2+1}}$

1. **An inclined plane**

A particle of mass m slides down a frictionless inclined plane under the influence of gravity. The inclination angle of the plane is θ. There is a velocity-dependent drag force on the particle of the form

$$F_{\mathrm{d}} = -kmv^2\hat{v}, \tag{1}$$

where $\vec{v}$ is the velocity of the particle, and k is a positive constant.

a) (10 points) The particle starts at rest. Calculate time required for it to move a distance D down the inclined plane. **Hint:** You may wish to use the table of integrals provided.

b) (10 points) Determine the power lost to the dissipative drag force when $D \to \infty$.

b) (5 points) Find the total energy lost to drag as a function of D. **Hint:** This is harder, and you will probably need to use the table of formulae.

For Calculations

For Calculations

2. **Finding a phase diagram**

Consider the motion of a point particle of mass m in a potential given by

$$U(x) = -U_0 \left[x^3 - ax^2\right], \tag{2}$$

where $U_0 > 0$ and $a > 0$ are constants.

a) (5 points) Sketch a graph of the potential above.

b) (5 points) Find the stable and unstable fixed points for the particle.

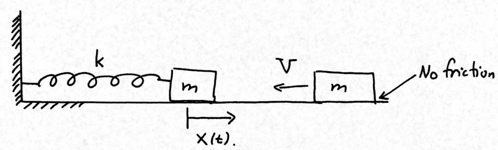
c) (5 points) What is the frequency of small oscillations about the stable fixed point.

d) (5 points) What is the maximum energy $E^\star$ of the particle if it undergoes bounded motion around the origin of the coordinates? What is the minimum energy?

e) (5 points) Sketch a phase space picture of typical trajectories of the particle for energies above and below the $E^\star$ (which you computed in part d).

For Calculations

For Calculations



3. **Colliding with an overdamped oscillator**

An overdamped harmonic oscillator moves along the x axis. It has a mass of m and is attached to a spring with spring constant k. There is a (large) damping force of the form:

$$F_{\mathrm{d}} = -bM\dot{x}. \tag{3}$$

where b is a positive constant and M is the **total mass of the block.** While the harmonic oscillator is at its rest length and stationary, another block of mass m slides **without friction** with speed V and collides with the mass attached to the spring. When the two blocks touch, they are instantly glued together. The two blocks oscillate together.

(a) (10 points) Find the equation of motion for the two stuck blocks.

(b) (15 points) Find the distance D that the two-block oscillator is compressed before rebounding.

For Calculations

For Calculations

4. **An undamped oscillator**

An undamped oscillator of mass m and with natural frequency ω_0 is subjected to a driving force given by

$$F(t) = \begin{cases} 0, & t < -\frac{\pi}{\omega_0} \\ F_0 \sin\left(\omega_0 t\right), & -\frac{\pi}{\omega_0} < t < \frac{\pi}{\omega_0} \\ 0, & t > \frac{\pi}{\omega_0} \end{cases} \tag{4}$$

a) (25 points) Find the equation of motion $x(t)$ of the oscillator for times $t > \frac{\pi}{\omega_0}$, when the force has returned to zero.

Hint: You look at the $\beta \to 0$ limit of the Green's function of the underdamped oscillator to determine the Green's function of the *undamped* oscillator.

For Calculations

For Calculations

For Calculations

Version 3
April 27, 2021