MAE 103 Elementary Fluid Mechanics

Kwitae Chong

Winter 2016

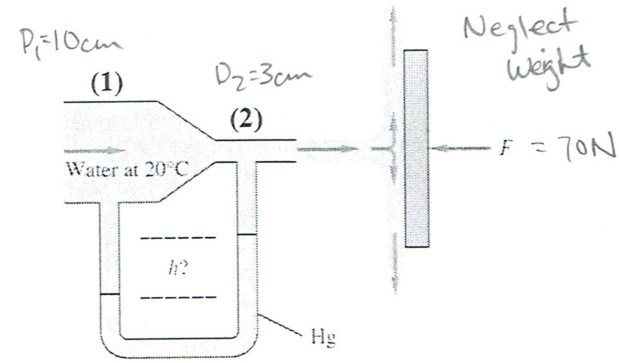
Midterm Examination

- The duration of this exam is 1 hour 50 minutes
- Write your name at the top of every sheet and your UID on the topmost sheet.
- Read each question carefully before answering.
- Be clear and concise with your answers, and please demonstrate every step. It is impossible to grade a solution if I have to guess your reasoning.
- Check units in your final answer, where possible.
- You are not allowed to use any materials or resources except a calculator and writing utensils.
- Absolutely **no collaboration or discussion** will be tolerated.
- If you find anything on the exam that is unclear, you may consult the instructor or TA.

1	40/40
2	13
3	43
total	96

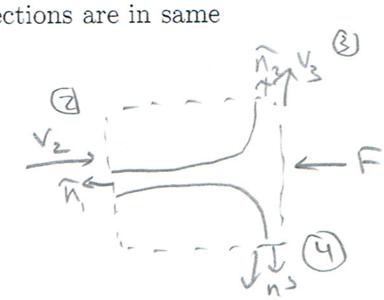
Name:

1. (40 points) Water flows through a circular nozzle, exits into the air as a jet, and strikes a plate, as shown. The force required to hold the plate steady is $70N$. The diameter of section (1) and (2) are $D_1 = 10cm$ and $D_2 = 3cm$, respectively. Assume steady, frictionless flow. Neglect the weight of the plate. $\rho_{H_2O} = 1000kg/m^3$, $\rho_{Hg} = 13600kg/m^3$ and $g = 9.8m/s^2$



(a) Estimate the velocities at sections (1) and (2). Assume that these sections are in same height.

(b) Estimate the mercury manometer reading h.



a) Mass conservation: $0 = \underbrace{\frac{d}{dt}\int_{cv} \cancel{\rho d\forall}}_{steady} + \int_{cs} \rho (\vec{v}_r \cdot \vec{n}) dA$

$\dot{m}_{in} = \dot{m}_3 + \dot{m}_4$

$\cancel{\rho} V_2 A_2 = A_3 \cancel{\rho} V_3 + \cancel{\rho} V_4 A_4$ incompressible

$V_2 = V_3 = V_4$

$A_2 = A_3 + A_4$ (external flow. area of flow does not change unless we force it to)

Momentum: $\Sigma F = \underbrace{\frac{d}{dt}\int_{cv} \cancel{\rho V dV}}_{steady} + \int_{cs} \rho V (v_r \cdot n) dA$

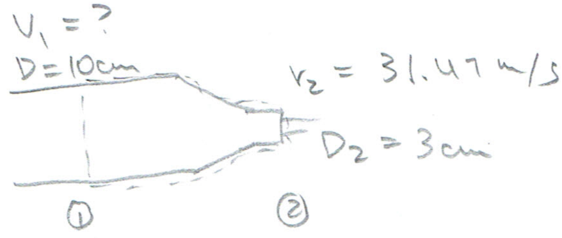
x: $\Sigma F_x = -F = \rho V_2 (-V_2) A_2 + 0$

$F = \rho V_2^2 A_2$

$V_2 = \sqrt{\frac{F}{\rho A_2}} = \sqrt{\frac{70N}{(1000\frac{kg}{m^3})(\frac{\pi}{4}(.03)^2)}} = \boxed{9.95 m/s = V_2}$

BACK

$\longrightarrow$



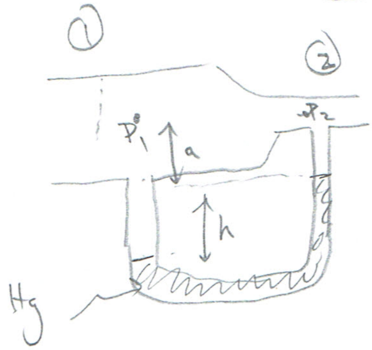
Mass Conservation: $\dot{m}_1 = \dot{m}_2$ (steady flow)

$$\cancel{\rho_1} V_1 A_1 = \cancel{\rho_2} V_2 A_2 \quad (\rho_1 = \rho_2, \text{incompressible})$$

$$V_1 \tfrac{\pi}{4}(.1)^2 = (9.95)\tfrac{\pi}{4}(.03)^2$$

$$\boxed{V_1 = 0.896 \, m/s}$$

b)



Bernoulli:

$$\frac{P_1}{\rho_1 g} + \frac{1}{2g}V_1^2 + \cancel{z_1} = \frac{P_2}{\rho_2 g} + \frac{1}{2g}V_2^2 + \cancel{z_2}$$

$z_1 = z_2$ $\qquad$ $z_1 = z_2$

$$\frac{1}{2\cancel{g}}V_1^2 - \frac{1}{2\cancel{g}}V_2^2 = \frac{P_2}{\rho \cancel{g}} - \frac{P_1}{\rho \cancel{g}}$$

$$\tfrac{1}{2}\rho_{H_2O}(V_1^2 - V_2^2) = P_2 - P_1$$

$$P_2 - P_1 = \tfrac{1}{2}(1000 \, kg/m^3)(.896^2 - 9.95^2) = \cancel{-49176} \; -49099$$

Manometry $\quad P_1 + \gamma_{H_2O}(\cancel{a}+h) - \gamma_{Hg} h - \cancel{\gamma_{H_2O} a} = P_2$

$$\rho_{H_2O} g h - \rho_{Hg} g h = P_2 - P_1$$

$$h = \frac{P_2 - P_1}{\rho_{H_2O} g - \rho_{Hg} g} = \frac{-49099}{(1000)(9.8) - 13600(9.8)} = \boxed{0.397 \, m = h}$$

Name:

13

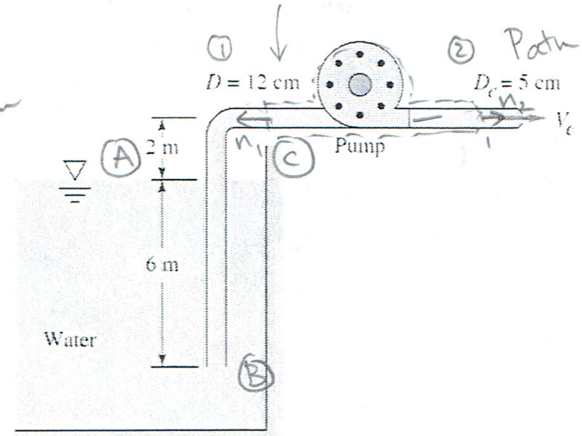
2. (15 points) When the pump draws 0.6 m^3/s of water from the reservoir, the total head loss is 5 m. The flow discharges through a nozzle to the atmosphere. Estimate the pump power delivered to the water. Water density and gravity acceleration are respectively, $\rho = 1000kg/m^3$ and $g = 9.8m/s^2$.

$P = ?$

$Q_1 = .6 m^3/s$

$H_{loss} = 5m$

$\dot{W}_{pump} = ?$



Mass: steady flow: $\dot{m}_1 = \dot{m}_2$

incompressible: $\rho V_1 A_1 = \rho V_2 A_2$

$Q_1 = V_2 A_2$

$V_2 = \frac{Q_1}{A_2} = \frac{.6 m^3/s}{\frac{\pi}{4}(.05)^2} = 305.58 m/s$

$V_1 A_1 = Q_1$

$V_1 = \frac{Q_1}{A_1} = \frac{.6}{\frac{\pi}{4}(.12)^2} = 53.05 m/s$

Pressure: $P_A + 6m(\gamma_{H_2O}) = P_B$

$P_A = P_\infty$ $P_B = 6\rho_{H_2O} g = 6(1000)(9.8) = 58800$

Bernoulli: B → C

$\frac{P_B}{\rho} + \boxed{\frac{1}{2}\cancel{V_B^2}} + g\cancel{z_B} = \frac{P_c}{\rho} + \frac{1}{2}V_c^2 + g z_c$ continuity −2

$V_B = V_C$ $z_B = 0$ $V_c = V_1$ $z_c = 6 + 2 = 8m$

$\frac{P_B}{\rho} = \frac{P_c}{\rho} + \frac{1}{2}V_1^2 + g(8)$

$\frac{58800}{1000} = \frac{P_c}{1000} + \frac{1}{2}\cancel{(53.05)^2} + (9.8)(8)$

$P_1 = P_c = -1426751.25$ (gauge)

$= 0 - 0$

BACK →

Energy: $\sum_{in} \dot{m}\left(\frac{P_1}{\rho} + \frac{1}{2}v_1^2 + gz_1\right) + \dot{W}_{pump} = \sum_{out} \dot{m}\left(\frac{P_2}{\rho} + \frac{1}{2}v_2^2 + gz_2\right)$

$+ \cancel{\dot{W}_{tub}}^{\,0} + \dot{E}_{loss}$

$$\frac{P_1}{\rho g} + \frac{1}{2g}v_1^2 + \cancel{z_1} + h_{pump} = \cancel{\frac{P_2}{\rho g}} + \frac{1}{2g}v_2^2 + \cancel{z_2} + h_{loss}$$

$z_1 = z_2$ $\quad$ $P_2 = P_{atm}$ $\quad$ $z_1 = z_2$

$$\frac{-1426751}{(1000)(9.8)} + \frac{1}{2(9.8)}(53.05)^2 + h_{pump} = \frac{1}{2(9.8)}(305.58)^2 + 5$$

$$h_{pump} = 4779.66 \text{ m}$$

$$\text{Power} = \dot{W}_{pump} = \dot{m} g h_{pump} = \rho v_1 A_1 g h_{pump}$$

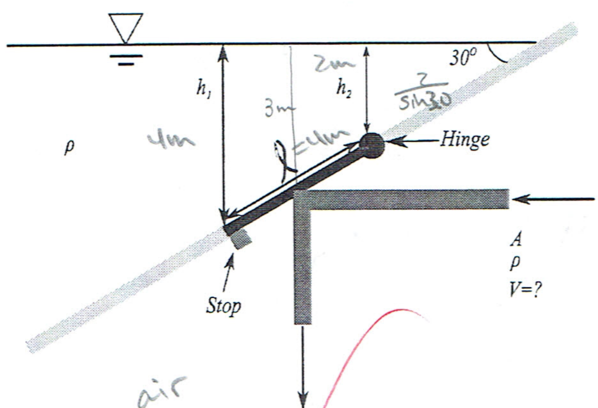
$$= (1000)(53.05)\left(\frac{\pi}{4}\right)(.12)^2(9.8)(4779.66)$$

$$\boxed{P_{pump} = 2.81 \times 10^7 \text{ Watts}}$$

Name:

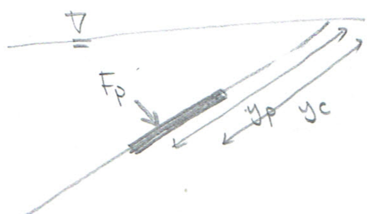
43

3. (45 points) Water with density $\rho = 1000kg/m^3$ is maintained in an inclined wall with hinged gate. The mass, length and width of rectangular gate are respectively, $m = 100kg$, $l = 4m$ and $w = 2m$. Water jet of density $\rho = 1000kg/m^3$ and area $A = 0.05m^2$ horizontally strikes the center of gate and is deflected vertically. The water jet maintains the same speed and area before/after it strikes the gate. $h_1 = 4m$, $h_2 = 2m$ and gravity acceleration is $g = 9.8m/s^2$. Determine the minimum speed of jet to open the gate.



$\sin 30 = \frac{h}{x}$

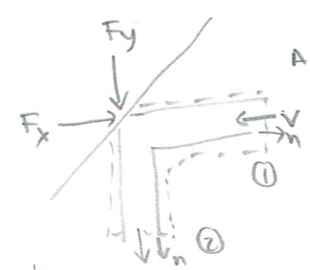
$x = \frac{h}{\sin 30}$



$y_c = \frac{3}{\sin 30} = 6m$ $\quad I_{xx,c} = \frac{1}{12} w l^3 = \frac{1}{12}(2)(4)^3 = \frac{32}{3}$

$F_p = \rho g y_c A \sin\theta = (1000)(9.8)(6)(4 \cdot 2)\sin 30 = 235200\ N$

$y_p = y_c + \frac{I_{xx,c}}{y_c A} = 6 + \frac{32/3}{6(4)(2)} = 56/9 = 6.22\ m$



$A = .05m^2$

Mass: $\dot{m}_1 = \dot{m}_2$ (steady)

$\rho A V_1 = \rho A V_2$ incompressible, external flow

$V_1 = V_2 = V$ (area does not change unless we force it to)

Momentum: $\Sigma F = \cancel{\frac{d}{dt}\int_{cv} \rho \vec{V} dV}_{\text{steady}} + \int_{cs} \rho \vec{V} (\vec{V}_r \cdot \hat{n}) dA$

$\Sigma F_x = F_x = \rho(-v)(-v)A + 0$

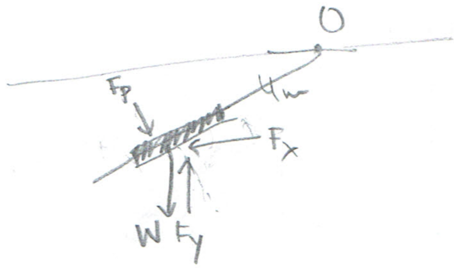
$= (1000)(v^2)(.05m^2) = 50v^2 = F_x$

4

$\Sigma F_y = -F_y = \rho(-v)(v)A =$

$F_y = \rho v^2 A = 1000(.05)v^2 = 50v^2 = F_y$

BACK →



Fx and Fy are equal and opposite forces from Fx, Fy on C.V.

-2

$$\circlearrowleft_{+} \sum M_0 = 0 = F_p(y_p - 4) + F_y \cos\theta (y_c - 4) + F_x \sin\theta (y_c - 4) + W \cos\theta (y_c - 4)$$

$$235200\left(\frac{56}{9}\right) - 50v^2 \cos 30\,(6) - 50v^2 \sin 30\,(6)$$

$$+ (100)(9.8)\cos 30\,(6) = 0$$

$$1468558 = v^2 (409.8)$$

$$\boxed{v = 59.86 \text{ m/s}}$$

This makes more physical sense, but both $\sum M_0 = \sum M_{Hinge} = 0$

$$\sum M_{Hinge} = F_p(y_p - 4) - F_y \cos 30 (y_c - 4) - F_x \sin 30 (y_c - 4) + W \cos\theta (y_c - 4)$$