MAE 101A
Statics and Strength of Materials
Midterm Exam 1

(Fall 2019)

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Problem 1	30
Problem 2	30
Problem 3	30
Total (100)	90

NAME [redacted]

UID : [redacted]

1. Please draw all your *free body diagrams* and write the associated *force* and *moment* balance equations clearly.
2. You are allowed one page of cheat sheet and a scientific calculator.
3. Attach additional sheets as needed and write your name on every sheet you use.

Problem 1 [30 points]

A uniform pipe cover of radius $r = 240$ mm and mass 30 kg is held in a horizontal position by the cable CD. The bearings are perfectly aligned. Assuming that the bearing at B does not exert any axial thrust, determine the tension in the cable and the reactions at A and B.

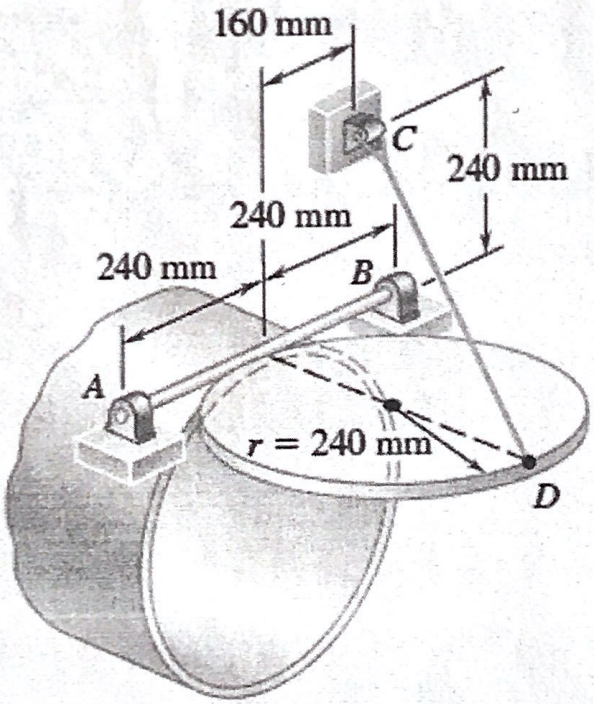

Figure 1: Problem 1

Problem 2 [30 points]

A stadium roof *truss* is loaded as shown. Determine the forces in members AB, AG, and FG.

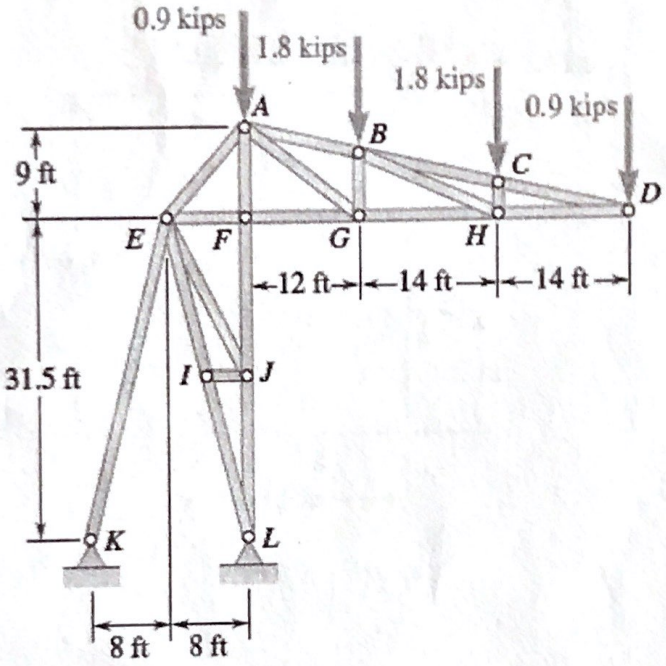

Figure 2: Problem 2

Problem 3 [40 points]

Two 9 in. diameter pipes (pipe 1 and pipe 2) are supported every 7.5 ft by a small frame like that shown. Knowing that the combined weight of each pipe and its contents is 30 lb/ft, and assuming friction-less surfaces, determine the components of reactions at A and G.

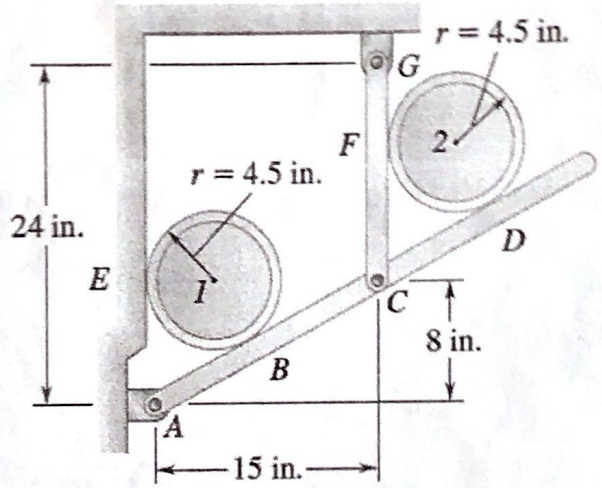

Figure 3: Problem 3

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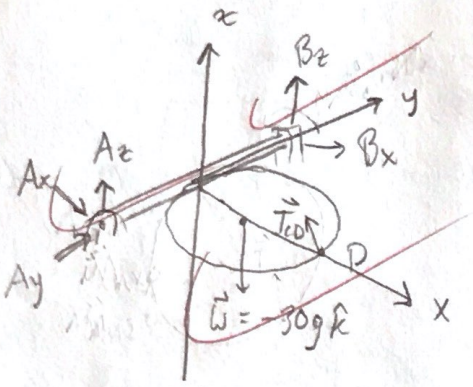
PROBLEM 1

FBD of entire system:



Coordinates of points:
A (0, −240, 0)
B (0, 240, 0)
C (0, 160, 240)
D (480, 0, 0)

We have 10 unknowns to solve for

Why 10 unknowns?

First, find direction of $\vec{T}_{CD}$:

$$\vec{r}_{DC} = (0-480)\hat{\imath} + (160-0)\hat{\jmath} + (240-0)\hat{k}$$
$$= -480\hat{\imath} + 160\hat{\jmath} + 240\hat{k}$$

$$\vec{e}_{DC} = \frac{\vec{r}_{DC}}{\sqrt{480^2 + 160^2 + 240^2}} = \frac{\vec{r}_{DC}}{560}$$
$$= -\frac{6}{7}\hat{\imath} + \frac{2}{7}\hat{\jmath} + \frac{3}{7}\hat{k}$$

$$\vec{T}_{CD} = |\vec{T}_{CD}|\,\vec{e}_{DC}$$

Considering FBD ①:

$$\Sigma F_x = T_{CDx} + B_x + A_x = 0 \quad (1)$$
$$\Sigma F_y = T_{CDy} + A_y = 0 \quad (2)$$
$$\Sigma F_z = T_{CDz} + B_z + A_z - 30g = 0$$
$$T_{CDz} + B_z + A_z = 30g \quad (3)$$

$\vec{e}_{CD} = -\frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{3}{7}\hat{k}$

PROBLEM 1 [continued]

$\Sigma M_{x\text{-axis}} = \vec{M}^{\,0}_{Ax} + \vec{M}^{\,0}_{Bx} - 240A_z + 240B_z = 0$ (4)

$\Sigma M_{y\text{-axis}} = 240(30g) - 480\,T_{CDz} = 0$ (5)

$-480\left(\frac{3}{7}T\right) = -240(30g)$

$T = 343.35\ N$

$\Sigma M_{z\text{-axis}} = \vec{M}^{\,0}_{Bz} + \vec{M}^{\,0}_{Az} + 240A_x - 240B_x + 480\,T_{CDy} = 0$ (6)

We get the following equations:

$T_{CDx} + B_x + A_x = 0$

$T_{CDy} + A_y = 0$

$T_{CDz} + B_z + A_z = 30g$

$-240A_z + 240B_z = 0$

$-480\,T_{CDz} = -240(30g)$

$240A_x - 240B_x + 480\,T_{CDy} = 0$

$$\begin{bmatrix} -\frac{6}{7} & 1 & 1 & 0 & 0 & 0 \\ \frac{2}{7} & 0 & 0 & 1 & 0 & 0 \\ \frac{3}{7} & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 240 & -240 \\ -480\left(\frac{3}{7}\right) & 0 & 0 & 0 & 0 & 0 \\ 480\left(\frac{2}{7}\right) & -240 & 240 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_{CD} \\ B_x \\ A_x \\ A_y \\ B_z \\ A_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 30g \\ 0 \\ -240(30g) \\ 0 \end{bmatrix}$$

$T_{CD} = 343.35\ N$ $\quad B_z = 73.575\ N$

$B_x = 245.25\ N$ $\quad A_z = 73.575\ N$

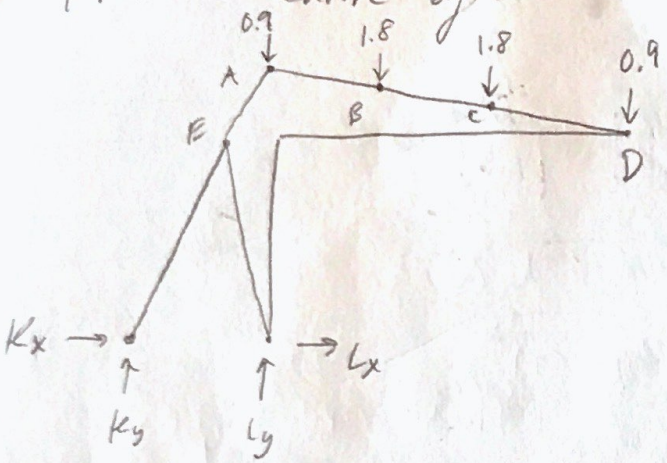
$A_x = 49.05\ N$

$A_y = -98.1\ N$

30/30

PROBLEM 2

FBD of entire system:



$$\Sigma M_K = 16L_y - 16(0.9) - 28(1.8) - 42(1.8) - 56(0.9) = 0$$

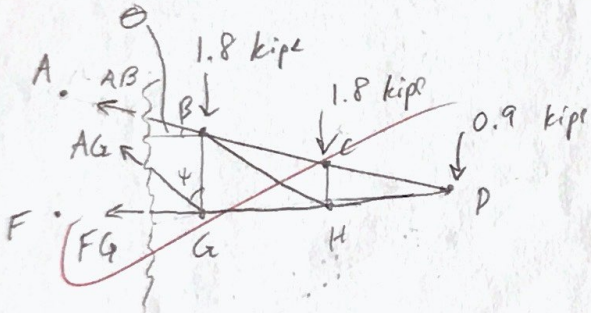
$$L_y = 11.925 \text{ kips}$$

$$\Sigma F_y = K_y + L_y - 0.9 - 1.8 - 1.8 - 0.9 = 0$$

$$K_y = -6.525 \text{ kips}$$

$$\Sigma M_x = L_x + K_x = 0$$

Using method of sections:



$$\theta = \tan^{-1}\left(\frac{9}{40}\right) = 12.6804^\circ$$

$$\psi = \tan^{-1}\left(\frac{12}{9}\right) = 53.1301^\circ$$

$$\Sigma M_A = -9FG - 12(1.8) - 26(1.8) - 40(0.9) = 0$$

$$FG = \frac{12(1.8) + 26(1.8) + 40(0.9)}{-9}$$

$$\boxed{FG = -11.6 \text{ kips (C)}}$$

$$\Sigma F_x = -(AG)\sin\psi - (AB)\cos\theta = FG \quad (1)$$

$$\Sigma F_y = -(1.8) - 1.8 - 0.9 + (AB)\sin\theta + (AG)\cos\psi = 0$$

$$(AB)\sin\theta + (AG)\cos\psi = 4.5 \quad (2)$$

PROBLEM 2 [continued]

$\psi = 53.1301^\circ$
$\theta = 12.6804^\circ$

From (1) and (2),

$$\begin{bmatrix} -\sin\psi & -\cos\theta \\ \cos\psi & \sin\theta \end{bmatrix} \begin{bmatrix} AG \\ AB \end{bmatrix} = \begin{bmatrix} FG \\ 4.5 \end{bmatrix}$$

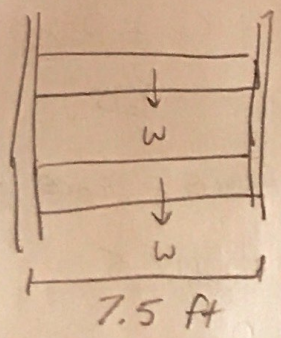
$AG = 4.50$ kips (T)

$AB = 8.20$ kips (T)

30/30

PROBLEM 3

Side view

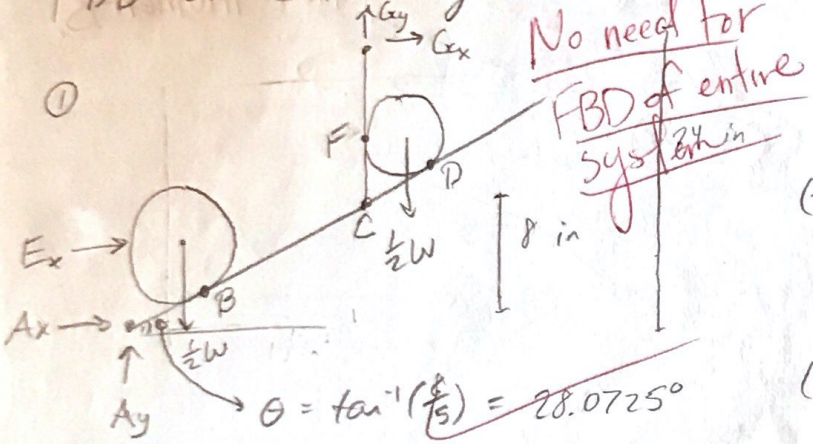


$W = (7.5\ ft)(30\ lb/ft) = 255\ lbs.$

each Frame supports $\frac{1}{2}W = 112.5\ lbs.$ (per pipe)

(per pipe)

FBD of entire system:



No need for FBD of entire system

Considering FBD ①:

(1) $\Sigma F_y = A_y + G_y - 2(\frac{1}{2}W) = 0$

$A_y + G_y = W$

(2) $\Sigma F_x = E_x + A_x + G_x = 0$

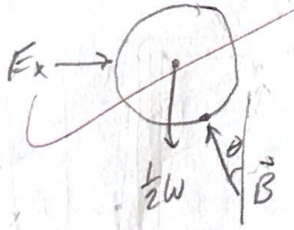
$\Sigma M_G = 24A_x + 16E_x - 15A_y + (10.5)(\frac{1}{2}W) - 4.5(\frac{1}{2}W) = 0$

(3) $24A_x + 16E_x - 15A_y = -675$

Variables in equations:

A_y
G_y
E_x
A_x
G_x
$B = 127.5\ lbs.$
C_x
C_y
$D = 127.5$
$F_x = 60$

Consider member 1:



$\Sigma F_x = -B\sin\theta + E_x = 0$

$\Sigma F_y = B\cos\theta - \frac{1}{2}W = 0$

$B = 127.5\ lbs.$

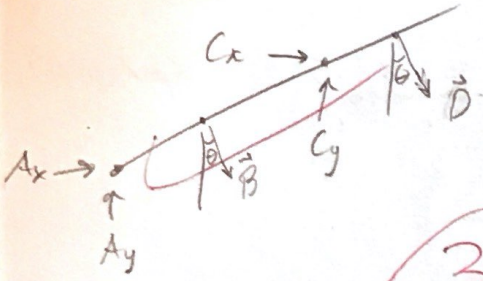
$E_x = B\sin\theta$

$E_x = 60\ lbs.$

PROBLEM 3 [continued]

$\theta = 28.0725°$

Consider member ABCD:



$\Sigma F_x = A_x + B\sin\theta + C_x + D\sin\theta = 0$

(4) $A_x + C_x + D\sin\theta = -60$

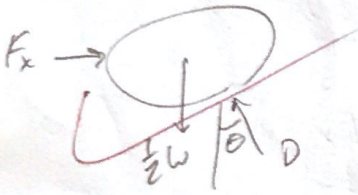
$\Sigma F_y = A_y + C_y - B\cos\theta - D\cos\theta = 0$

(5) $A_y + C_y - D\cos\theta = 112.5$

30/40

Member 2:

These are forces not moments.



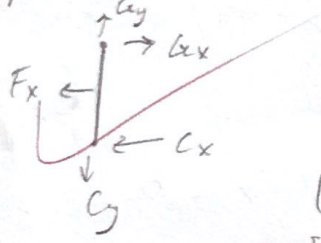
$\Sigma M_x = F_x - D\sin\theta = 0$

$\Sigma M_y = D\cos\theta - \frac{1}{2}W = 0$

$D = 127.5$

$F_x = D\sin\theta = 60$

Member GFC:



$\Sigma F_x = G_x - C_x - F_x = 0$

$G_x - C_x = 60$ (6)

$\Sigma F_y = G_y - C_y = 0$ (7)

Using equations 1-7:

A_y
G_y
F_x

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ -15 & 0 & 16 & 24 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} A_y \\ G_y \\ F_x \\ A_x \\ C_x \\ C_y \\ G_x \end{bmatrix} = \begin{bmatrix} 255 \\ 0 \\ -675 \\ -120 \\ 225 \\ 60 \\ 0 \end{bmatrix}$$

ran out of time, but
Solving the matrix will give answer

A_x
C_x

[redacted]

MAE 101 Cheat Sheet

Moment about a line, where $e \parallel L$:

$$M_L = (\vec{e} \cdot [\vec{r} \times \vec{F}])\vec{e}$$

Moments: Moment $\vec{M}$ due to a couple is same about any point

$|\vec{M}| = d|\vec{F}|$, where d = perp dist. btw lines of action of forces

TRUSSES

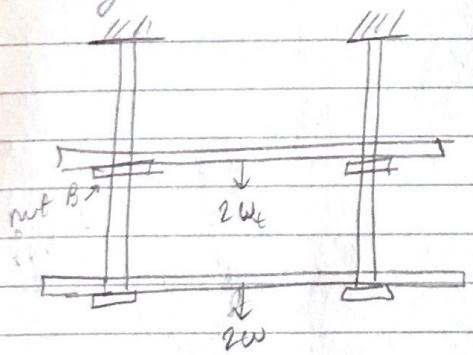
1) Find reaction forces @ supports
2) either joint method or method of sections

FRAMES

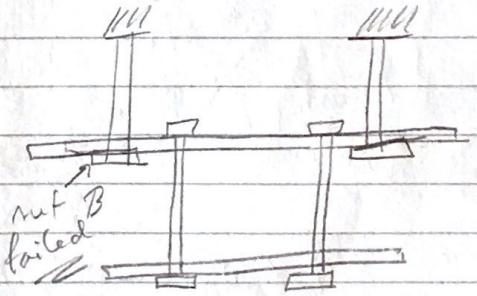
1) Find reactions at supports
2) Explode each member individually

HYATT BRIDGE

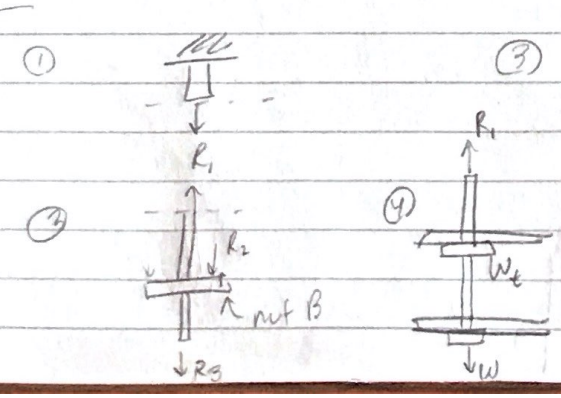
original:



As-Built:



ORIGINAL



(1) $\sum F_y = R_1 - R_2 - R_3$

$R_1 = R_2 + R_3$

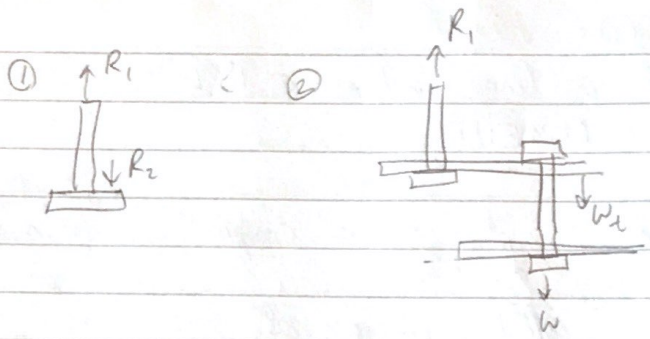
(3) $R_3 - W = 0$

$R_3 = W$

(4) $R_1 = W + W_t$

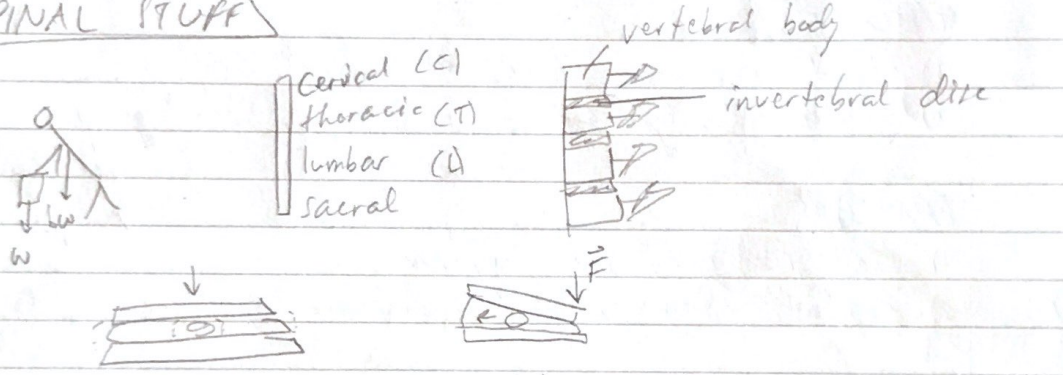
$W + W_t = R_2 + W$

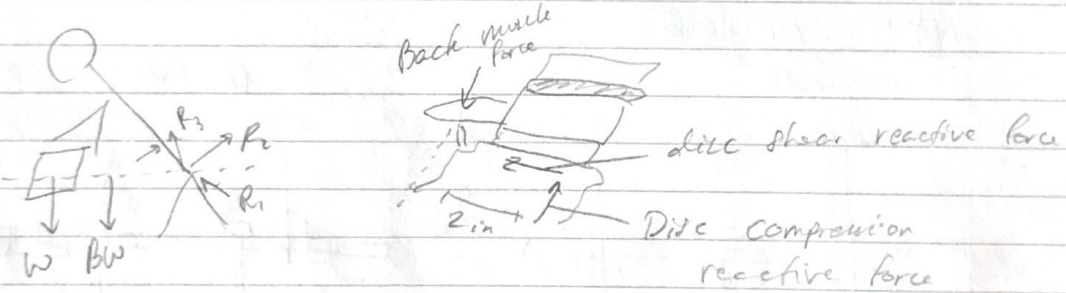
$R_2 = W_t$



(1) $R_1 = R_2$ (2) $R_1 = W_t + W \rightarrow R_2 = W_t + W$

SPINAL STUFF





↑ compression strength in lower vertebral bodies