Problem 1)

$P(\text{children all choose different \#}) = \frac{200!}{160!(200)^{40}} \approx 0.01521$

$P(\text{2 or more children choose same \#}) = 1 - P(\text{children all chose different \#})$

$\approx 1 - 0.01521 \approx 0.98479$

$\boxed{98.479\%}$

Problem 2)

$P(\text{at least 2 sixes}) = 1 - P(\text{no sixes}) - P(\text{exactly one six})$

$= 1 - \left(\frac{5}{6}\right)^n - n \cdot \left(\frac{5}{6}\right)^{n-1} \cdot \frac{1}{6}$

n is # of dice needed to have 50% probability of rolling 2 or more sixes.

For $n=9$: $P(\text{at least 2 sixes}) = 1 - \left(\frac{5}{6}\right)^9 - 9 \cdot \left(\frac{5}{6}\right)^8 \cdot \frac{1}{6}$

≈ 0.457

For $n=10$: $P(\text{at least 2 sixes}) = 1 - \left(\frac{5}{6}\right)^{10} - 10 \cdot \left(\frac{5}{6}\right)^9 \cdot \frac{1}{6}$

≈ 0.515

We need at least $\boxed{10 \text{ dice}}$ to have a 50% probability of rolling 2 or more sixes.

Problem 3)

z-score for 95 percentile: 1.645

$z = \frac{x - \mu}{\sigma}$ $\quad 1.645 = \frac{x - 5.1}{0.6}$ $\quad x = \boxed{6.087 \text{ mmol/L}}$

Problem 4)

Assume we invest a fraction f in A and a fraction $1-f$ in B. We wish to choose a f such that we minimize $\sigma(fA+(1-f)B)$ where A and B are random variables giving the return from funds A & B.

It follows that minimizing σ^2 is the same as minimizing σ

$$\sigma^2(fA+(1-f)B) = \sigma^2(fA) + \sigma^2((1-f)B) + 2\,\mathrm{cov}(fA,(1-f)B)$$

$$= f^2\sigma^2(A) + (1-f)^2\sigma^2(B) + 2f(1-f)\,\mathrm{cov}(A,B)$$

$$= (\sigma_A^2 + \sigma_B^2 - 2\sigma_A\sigma_B\rho_{AB})f^2 + (2\sigma_A\sigma_B\rho_{AB} - 2\sigma_B^2)f + \sigma_B^2$$

$$= af^2 + bf + c \text{ for some } a,b,c$$

Since $\rho_{AB} < 0$, it follows $a > 0$ and thus this is a parabola that opens upwards and has a minimum.

$$\text{Min @ } f = \frac{-b}{2a} = \frac{\sigma_B^2 - \sigma_A\sigma_B\rho_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_A\sigma_B\rho_{AB}}$$

Verify $0 \le f \le 1$ since can't invest negative amount or more than 100%. Since $\rho_{AB} < 0$, it follows that $f > 0$ and since $\sigma_A^2 > 0$, it follows $f < 1$