Version 2

MATH 61: INTRODUCTION TO DISCRETE STRUCTURES
MIDTERM #1

INSTRUCTOR: SPENCER UNGER

Name: Algan Binggi Rustinya

ID # 604 670 111

Section 1A

Good Luck! Be sure to justify your answers!

No calculators, books or notes are allowed.

Problem	Points	Score
1	15	15
2	15	7
3	15	8
4	15	5
5	15	13
Total	75	48

Be careful, there are problems on both sides of the paper!

Date: April 18, 2016.

(1) (15 points) Consider the set A of points on the circle of radius 1 centered at the origin. Define a relation R on A by $(p, q) \in R$ exactly when q can be obtained by moving p some multiple of 45 degrees around the circle *in either direction!*. So for example $(1, 0)$ is related to $(-\sqrt{2}, \sqrt{2})$ since we can move from the first point to the second by moving counterclockwise by 45 degrees *three times*. However, $(0, 1)$ is not related to $(.5, \sqrt{1 - .5^2})$, because no amount of rotating by 45 degrees will move us between the points.

(a) Show that R is an equivalence relation.

For R to be an equivalence relation, we need to show that R is reflexive, symmetric, and transitive

Prove Reflexive:

let p be some arbitrary point around the circle the exact point p can be obtain by moving it around the circle for 1 cycle or 360 degree in either direction and 360 is a multiple of 45 ($45 \times 8 = 360$). Therefore $(p, p) \in R$, thus R is reflexive

Prove Symmetric:

If $(p, q) \in R$ then we can find q from p by moving multiple of 45 degree either direction. Thus we can also find p from q by moving the opposite direction thus if $(p, q) \in R$ then $(q, p) \in R$. and R is symmetric

Prove transitive:

If $(p, q) \in R$ and $(q, r) \in R$, then. Since q can be obtain from p with a multiple of 45 and r can be obtain from q with a multiple of 45. then by adding these 2 direction, we can get from p to r with a multiple of 45, thus $(p, r) \in R$, and R is transitive

Therefore R is equivalence relation

(b) List the elements of $[(1, 0)]$.

$\{ (1,0), (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}), (0,1), (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}), (-1,0), (-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}), (0,-1), (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) \}$

15

(2) (15 points; each of (a), (b) and (c) are weighted equally) Answer the following questions.

(a) Let $A = \{a, b, c\}$, $B = \{1, 2, 3\}$ and $C = \{b, c, 2, 3\}$. Determine each of the following. If it is a set, then write the set by listing the elements. If it is a number, then write which one. If it is a statement, then write true or false.

(i) $|B \cup C|$ 5

(ii) $A \times B$ $\{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3), (c,1), (c,2), (c,3), (1,a), (2,a), (3,a), (1,b), (2,b), (3,b), (1,c), (2,c), (3,c)\}$

(iii) $(A \cup B) - C$ $\{a, 1\}$

(iv) $C \cap (B \times B)$ $\emptyset$

(v) $(A \cap B) \subseteq C$ True

✓

(b) Let A_1 and A_2 be sets. Find *disjoint* sets B_1 and B_2 such that $A_1 \cup A_2 = B_1 \cup B_2$.

Let $A_1 = \{1,2\}$ and $B_1 = \{1,2\}$

$A_2 = \{2,3\}$ $B_2 = \{3\}$

thus

0/5

$A_1 \cup A_2 = B_1 \cup B_2$ and $B_1 \cap B_2 = \emptyset$ or disjoint

(c) Prove or disprove the following statement. For any sets A, B and X, if $A \subseteq B \subseteq X$, then $X - A \subseteq X - B$.

False. since

$X - A \subseteq X - B$ Â

$-A \subseteq -B$ and ? 2/5

$B \subseteq A$.

(3) (15 points) Consider the numbers between 10000 and 99999 inclusive. **To receive full credit you must explain which counting principles you use and how you are using them.**

(a) How many of them do not repeat a digit?

use multiplication principle.

$9 \times 9 \times 8 \times 7 \times 6 = 9 \times \frac{9!}{5!}$

(b) How many of them do not contain a 5 or a 6?

Using addition principle we breakdown to 3 cases:

case ①: do not contain 5, use multiplication principle

$8 \times 9 \times 9 \times 9 \times 9 = 8 \cdot 9^4$

case ②: do not contain 6, use multiplication principle.

+3

$8 \times 9 \times 9 \times 9 \times 9 = 8 \cdot 9^4$

case ③: do not contain 5 x 6 use multiplication principle

$7 \times 8 \times 8 \times 8 \times 8 = 7 \cdot 8^4$

So there are ① + ② - ③ $= 8 \cdot 9^4 + 8 \cdot 9^4 - 7 \cdot 8^4$

(c) How many of them contain exactly one 2?

use multiplication principle

$5! \times 1 \times 10 \times 10 \times 10 \times 10 = 120,0000.$

(4) (15 points) Define the following sets:

(a) Let X be the set of strings with alphabet $\{0, 1\}$.

(b) Let Y be the set of strings with alphabet $\{a, b\}$.

(c) Let Z be the set of strings with alphabet $\{0, 1, a, b\}$.

Define a function $F : X \times Y \to Z$ by $F(\alpha, \beta) = \alpha.\frown \beta$. Recall that $\alpha \frown \beta$ is the concatenation of α and β.

(a) Is F injective? Justify your answer.

Yes F is injective since. - X and Y are disjoint and. $\alpha \in X$ and $\beta \in Y$

$F(\alpha_1, \beta_1) = F(\alpha_2, \beta_2)$.

$\alpha_1 \frown \beta_1 = \alpha_2 \frown \beta_2$.

thus $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$.

why?

4

(b) Is F surjective? Justify your answer.

~~Yes F is surjective because every element in Z are either in X or in Y, such that. $X \cup Y = Z$.~~

~~therefore for every element in Z there exist an element of (α, β) in (X, Y) such that $f(\alpha, \beta) = \alpha \frown \beta$~~

0

(c) Recall that the range of F is the set $\{z \in Z \mid \text{there is } (x, y) \in X \times Y \text{ such that } F(x, y) = z\}$. Show that $X \cup Y$ is a subset of the range of F.

~~$X \cup Y = \{0, 1, a, b\}$~~

~~$F = Z = \{0, 1, a, b\}$~~

~~thus. $X \cup Y = Z$ and $X \cup Y \subseteq F$.~~

1

su

(5) (15 points) Prove by induction that for all $n \geq 4$, $2^n \geq n^2$.

Base case : $n = 4$

$2^4 \geq 4^2$

$16 \geq 16$ ✓

Induction step: ~~Assume~~ Assume true that $2^n \geq n^2$ for all $n \geq 4$

prove $2^{n+1} \geq (n+1)^2$.

$2^n \cdot 2 \geq n^2 + -2n + 1$

$n^2 \cdot 2 \geq n^2 + 2n + 1$

$n^2 - 2n - 1 \geq 0$

conclusion?

13