Discrete Structures
Math 61, Winter 2015 — Schaeffer
Midterm Exam 1

Name and Bruin ID: Kevin Huynh [redacted]

In LARGE CAPITALS, the first 3 letters of your last/family name:

HUY

Circle your TA. If you do not know your TA's name, you must speak with Professor Schaeffer when you hand in your exam so he can look it up.

Zhu (A,B) Rosenbaum (C,D) Zhang (E,F)

Instructions: Complete all problems. Notes and electronics are not permitted. Good luck!

Problem	Notes	Grade
1	XO	2
2	✓	3
3	✓	4
4	✓	7
5	✓	4
6	✓	4
7	✓	7
8	✓	4
Total		35

$\mathbb{Z}$ - all integers

1. Consider the sets S, T, and U defined below:

$$S = \{n \in \mathbb{Z} : \text{there is } m \in \mathbb{Z} \text{ such that } n = m^2\} \quad \{0, 1, 4, 9, 16, \ldots\}$$
$$T = \{r \in \mathbb{R} : \text{there is } x \in \mathbb{R} \text{ such that } r = \sqrt{x}\} \quad [0, \infty) \quad \text{all } \mathbb{R} \geq 0$$
$$U = \{r \in \mathbb{R} : -1 < r < 1 \text{ and } r \neq 0\} \quad (-1, 1), \text{ but } r \neq 0$$

Which of the following statements are true about these sets?

Circle all correct answers.

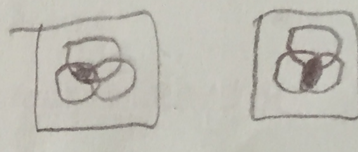
a. S and U are disjoint.

b. $T \cap U = U$

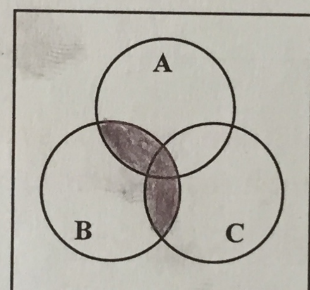
c. $S \subseteq T$

d. $S \cap T = S$

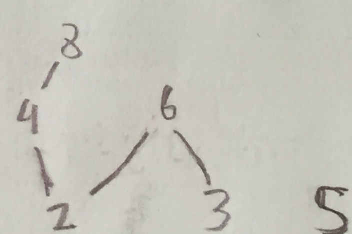
e. $T \cup U \subseteq \{r \in \mathbb{R} : r + 1 > 0\}$



2. In the Venn diagram below, shade the region corresponding to $(A \cap B) \cup (B \cap C)$.



3. Draw a Hasse diagram for the partial order of *divisibility* on the set $S = \{2, 3, 4, 5, 8, 6\}$.

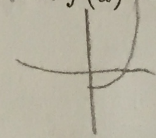


XO	✓	✓

4. For a.–c., if the function f is *injective*, circle it.

If the function f is not *injective* then in the blank spaces below that function, write down two elements a, b of the domain such that $a \neq b$ but $f(a) = f(b)$.

a. $f : [0, \infty) \to \mathbb{R}$ given by $f(x) = x^2 - 1$.



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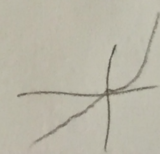
b. $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(n) = \begin{cases} n & \text{if } n < 0 \\ n^2 & \text{if } n \geq 0 \end{cases}$



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c. $f : \mathcal{P}(\{1, 2, 3\}) \to \mathcal{P}(\{1, 2, 3\}) : S \mapsto S - \{1\}$.

$\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$a = \emptyset$ $b = \{1\}$

5. State the *inclusion–exclusion principle*: If A and B are finite sets, then...

$|A \cup B| = |A| + |B| - |A \cap B|$. the cardinality of $A \cup B$ = the cardinality of A + the cardinality of B − the cardinality of $A \cap B$

$S = \{0,1\} \{0,2\} \{0,3\} \{1,1\} \{1,2\} \{1,3\} \{2,1\} \{2,2\} \{2,3\} \{3,1\} \{3,2\} \{3,3\}$

6. Let $S = \{0, 1, 2, 3\} \times \{1, 2, 3\}$ and let $\sim$ be the relation on S defined by

$$(a, b) \sim (c, d) \quad \text{if} \quad a = c$$

List all the elements of the $\sim$-equivalence class $[(0, 3)]$.

$\sim$ Is an eq. relation (you do not have to prove this.)

$[(0,3)] = \{(0,1), (0,2), (0,3)\}$

$A \sqsubset A$, $A \sqsubset B$, $B \sqsubset A$, $B \sqsubset B$, $B \sqsubset C$,
$C \sqsubset B$, $C \sqsubset C$, $D \sqsubset D$

7. One way to represent a relation is using a table.

Let $S = \{A, B, C, D\}$. We define a relation $\sqsubset$ on S by writing an X in the <u>row</u> for X and the <u>column</u> for Y in the table if (and only if) $X \sqsubset Y$:

Y

X

	A	B	C	D
A	X	X		
B	X	X	X	
C		X	X	
D				X

For example, reading the first row: $A \sqsubset A$ and $A \sqsubset B$, but $A \not\sqsubset C$ and $A \not\sqsubset D$.

Using the table above, answer the following questions about the relation $\sqsubset$.

In a.–c. you do not have to justify "Yes."

a. Is the relation $\sqsubset$ *reflexive*? If you answered "No," why not?

Yes

b. Is the relation $\sqsubset$ *symmetric*? If you answered "No," why not?

Yes

c. Is the relation $\sqsubset$ *transitive*? If you answered "No," why not?

No, $A \sqsubset B$ and $B \sqsubset C$, but $A \not\sqsubset C$. The definition of transitivity is that for xRy and yRz, xRz

8. Prove the following statement by *mathematical induction*: For all integers $n \geq 1$,

$$\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$$

Try to keep your proof as organized as you possibly can!

base case: $n=1$ $\quad \sum_{k=1}^{1} k(k+1) = 1(1+1) = 1(2) = 2$

$$= \frac{1(1+1)(1+2)}{3} = \frac{1(2)(3)}{3} = 2$$

base case true ✓

inductive step: Assume $P(n)$ is true for some fixed n, prove $P(n+1)$ holds. Prove $\sum_{k=1}^{n+1} k(k+1) = \frac{(n+1)(n+2)(n+3)}{3}$

$$\sum_{k=1}^{n+1} k(k+1) = \sum_{k=1}^{n} k(k+1) + [(n+1)(n+2)]$$

$$= \frac{n(n+1)(n+2)}{3} + (n+1)(n+2) = \frac{n(n+1)(n+2)}{3} + \frac{3(n+1)(n+2)}{3}$$

$$= \frac{n(n+1)(n+2) + 3(n+1)(n+2)}{3} = \frac{(n+1)(n+2)[n+3]}{3}$$

$$= \frac{(n+1)(n+2)(n+3)}{3}.$$ $P(n+1)$ is proven true, so ✓

statement holds for all integers $n \geq 1$ by ✓ proof of mathematical induction

✓	✓