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MATH 61 - QUIZ 6

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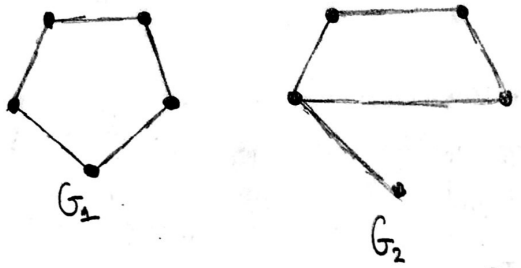
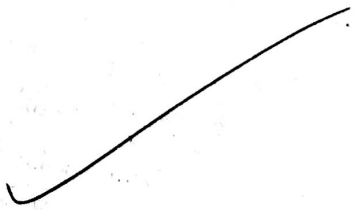
Exercise 0.1. Give examples of two graphs G_1 and G_2 that are both connected graphs with 5 vertices and exactly one simple cycle and yet G_1 and G_2 are not isomorphic.

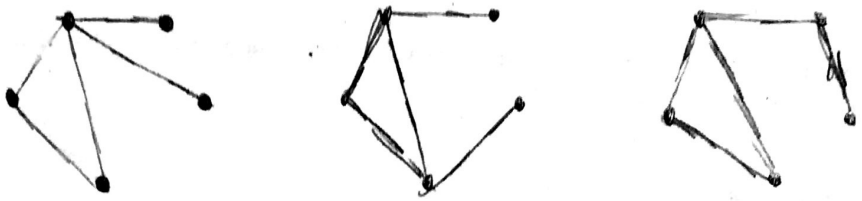


These are not isomorphic because G_2 has a vertex of degree 3 whereas all vertices in G_1 are of degree 2.



Date: November 26, 2018.

Exercise 0.2. Show that any connected graph having exactly 5 vertices and exactly one simple cycle must be planar.



Case 1: The simple cycle contains 3 vertices.
Clearly, the cycle is planar. The two remaining vertices must be attached with one edge each to the cycle (and no edge between them), or one is attached to the cycle and the other is connected to the first as the only edges to avoid creating any new cycles. These are all planar.

Case 2: The simple cycle contains 4 vertices. These 4 vertices cannot have any other edges between them, otherwise they would form a simple cycle of length 3, so they are clearly planar. The 5th vertex can only be attached by an edge to one of the four vertices (more edges would form a cycle), so the graph is planar.

Case 3: The simple cycle contains all 5 vertices. The graph must be planar because the 5 vertices cannot have any additional edges between them because that would form smaller cycles.

Since a cycle must consist of at least three vertices, any connected graph with 5 vertices and exactly 1 simple cycle must be planar. ∎