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MATH 61 - QUIZ 5

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Exercise 0.1. The polynomial $(x+4)^{100}$ may be expanded to $a_{100}x^{100} + a_{99}x^{99} + \ldots + a_1 x + a_0$, where each a_i is a real number. In this expansion, what is the coefficient a_{50} of x^{50}? *Hint*: binomial theorem.

By the binomial theorem, $\binom{100}{50} x^{50} 4^{50} \Rightarrow \boxed{\binom{100}{50} \cdot 4^{50}}$

$$\binom{100}{100} x^{100} 4^0 + \binom{100}{99} x^{99} 4^1 + \binom{100}{98} x^{98} 4^2 + \cdots + \binom{100}{1} x^1 4^{99} + \binom{100}{0} x^0 4^{100}$$

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Exercise 0.2. Abigail the dog-lover loves dogs but doesn't feel ready to own one yet. So she fosters a new dog each month for her local dog shelter before they find a permanent home. Her local dog shelter has dogs of 5 different breeds. How many months must Abigail foster dogs before she is guaranteed to have fostered 5 dogs of the same breed?

$5 \cdot 4 = 20$ (Foster 4 dogs of each breed)

21 months (Must choose a 5th dog of a breed)

$\left\lceil \frac{21}{5} \right\rceil = 5$ Pigeonhole Principle. At least 1 dog breed must have at least 5 dogs fostered by Abigail.