Instructions:

- You have from Monday May 17 at 12:00am to Monday May 17 11:59pm Pacific Time to solve this exam.
- Scan or type your solutions and upload them to Gradescope. You should submit readable scans, and not pictures of your solutions (you can use a scanner app on your phone, for instance). Please make sure to match the problems on the exam template with the respective parts in your solutions.
- This exam is open book, and you are allowed to use the textbook, and all resources from the lecture, or similar resources.
- You are not allowed to ask for help from others, nor give help to others taking this exam. Students suspected of academic dishonesty may be reported to the Dean of Students.
- Show your work. Full points are only given for correct answers *with* adequate justification. A correct final answer with missing or substantially incorrect justification will not merit full points on a problem.
- On any question asking you to calculate a number, you may leave your final answer either in combinatorial notation (using factorials and combinations, etc.) or a precise numerical value.
- All graphs are assumed to be simple.

Code of honour

Academic integrity is of the uttermost importance. By taking part in this evaluation, you are accepting the following code of honor:

I certify on my honor that I have neither given nor received any help, or used any non-permitted resources, while completing this evaluation.

Problem 1. Let's suppose that our class has exactly 200 students.

1. There is an urn at the front of the classroom full of yellow, red, and green balls. Each student comes up and 6 times draws a ball and puts it back, and then writes down how many each of yellow, red, and green balls they drew. Show that there must be at least 8 students in the class who drew exactly the same number of yellow, red, and green balls.

2. Now suppose instead of drawing balls, each student writes down a string of length 6 in the alphabet $\{A, B, C\}$. How many students must be there be in the class to be guaranteed that at least two students write down the same string?

Problem 2. Suppose $\{s_n\}_{n=0}^{\infty}$ is a sequence satisfying the following:

- $2s_n - 14s_{n-1} + 24s_{n-2} = 0$ for all $n \geq 2$.
- $s_0 = s_1 = 1$.

What is $\{s_n\}_{n=0}^{\infty}$? Your answer should be an explicit formula for s_n in terms of n.

Problem 3. If $G = (V, E)$ is a simple graph, we define the *complement graph* $G' = (V, E')$ to be the simple graph with the same vertices as G, but such that, for all distinct $v, w \in V$, v and w are adjacent in G' if and only if they are *not* adjacent in G. In other words, to form G', you put edges where there were not edges in G, and remove all edges that were in G.

1. Suppose G is a simple connected graph with 6 vertices. Give an example of such a G such that the complement graph G' of G is also connected (a picture is sufficient as long as the graph is clearly depicted by your picture).

2. Suppose G is a simple graph with 6 vertices which has an Euler cycle. Is it possible for the complement graph G' of G to have an Euler cycle as well? Justify your answer.

Problem 4. Let $X = \{1, 2, ..., 10\}$ and let V be the set of all subsets of X with cardinality exactly 2. Let G be a simple graph with vertex set V such that there is an edge between v and w from V if and only if $|v \cap w| = 1$, i.e. if v and w have exactly one element in common.

1. Is G connected?
2. What is the degree of the vertex $v = \{1, 2\}$?
3. Does G have an Euler cycle?