Instructions:

- You have from Wednesday April 21 at 12:00am to Wednesday April 21 11:59pm Pacific Time to solve this exam.
- Scan or type your solutions and upload them to Gradescope. You should submit readable scans, and not pictures of your solutions (you can use a scanner app on your phone, for instance). Please make sure to match the problems on the exam template with the respective parts in your solutions.
- This exam is open book, and you are allowed to use the textbook, and all resources from the lecture, or similar resources.
- You are not allowed to ask for help from others, nor give help to others taking this exam. Students suspected of academic dishonesty may be reported to the Dean of Students.
- Show your work. Full points are only given for correct answers *with* adequate justification. A correct final answer with missing or substantially incorrect justification will not merit full points on a problem.
- On any question asking you to calculate a number, you may leave your final answer either in combinatorial notation (using factorials and combinations, etc.) or a precise numerical value.

Code of honour

Academic integrity is of the uttermost importance. By taking part in this evaluation, you are accepting the following code of honor:

I certify on my honor that I have neither given nor received any help, or used any non-permitted resources, while completing this evaluation.

Problem 1. Prove by induction that, for all natural numbers n, the number of strings of length n in the alphabet $X = \{a, b, c, d\}$ is 4^n.

Problem 2. Suppose $X = \{x_1, x_2, x_3, x_4\}$. Let R be a relation on X defined by

$$R = \{(x_1, x_2), (x_4, x_3)\}.$$

1. Give an example of a relation S on X such that $R \subseteq S$ and S is an equivalence relation.
2. Give an example of a relation T on X such that $R \subseteq T$ and T is a partial order.

You must justify that your example works for both (1) and (2).

Problem 3. Let $\mathcal{S}$ denote the set of sequences with domain $\{1, 2, 3\}$ and whose codomain is also $\{1, 2, 3\}$.

1. How many sequences in $\mathcal{S}$ are increasing?

2. How many sequences in $\mathcal{S}$ are non-increasing?

3. What is the cardinality of the set of sequences in $\mathcal{S}$ that are are non-increasing or non-decreasing (inclusive)? That is, what is

$$|\{s \in \mathcal{S} : s \text{ is non-increasing or non-decreasing}\}|?$$

Problem 4. You own 4 hamsters and 5 gerbils and decide to march them around in a rodent parade. You will line them up single file, with all the hamsters together and all the gerbils together (so there can be no gerbil between two hamsters nor can there be a hamster between two gerbils in the lineup), though your parade may begin either with the hamsters or with the gerbils. How many ways are there for you line up your hamsters and gerbils for the parade?