Problem 1. Prove by induction that, for all natural numbers n, the number of strings of length n in the alphabet $X = \{a, b, c, d\}$ is 4^n.

Problem 2. Suppose $X = \{x_1, x_2, x_3, x_4\}$. Let R be a relation on X defined by

$$R = \{(x_1, x_2), (x_4, x_3)\}.$$

1. Give an example of a relation S on X such that $R \subseteq S$ and S is an equivalence relation.
2. Give an example of a relation T on X such that $R \subseteq T$ and T is a partial order.

You must justify that your example works for both (1) and (2).

Problem 3. Let $\mathcal{S}$ denote the set of sequences with domain $\{1, 2, 3\}$ and whose codomain is also $\{1, 2, 3\}$.

1. How many sequences in $\mathcal{S}$ are increasing?

2. How many sequences in $\mathcal{S}$ are non-increasing?

3. What is the cardinality of the set of sequences in $\mathcal{S}$ that are are non-increasing or non-decreasing (inclusive)? That is, what is

$$|\{s \in \mathcal{S} : s \text{ is non-increasing or non-decreasing}\}|?$$

Problem 4. You own 4 hamsters and 5 gerbils and decide to march them around in a rodent parade. You will line them up single file, with all the hamsters together and all the gerbils together (so there can be no gerbil between two hamsters nor can there be a hamster between two gerbils in the lineup), though your parade may begin either with the hamsters or with the gerbils. How many ways are there for you line up your hamsters and gerbils for the parade?