Mid 2 - q1b

Part 1.

a.) we first choose 7 of the 10 books using combination

Thus we have $C(n,r) = \binom{10}{7}$

$$= \frac{10!}{(7!(10-7)!)} = \frac{10\cdot 9\cdot 8}{3!} = \frac{720}{6} = 120$$

Thus there are 120 ways to choose 7 books. Since the books are all the same, there is only 1 way to arrange the books in a line which can be shown by $\frac{P(7,7)}{7!} = 7! \cdot \frac{1}{7!} = 1$

thus by multiplication principle the number of ways to choose 7 books AND arrange them in a line is $\boxed{120}$.

b) The number of ways to choose 7 books and place them inside a backpack is just the combination $C(10,7) = \binom{10}{7}$ which from part a.) we know to be 120 thus

there are 120 ways to choose 7 books from 10 to place in a backpack c.)

$$4 \cdot 4 \cdot 3 \cdot 4 \cdot 3 \cdot 3 \cdot 4 \cdot 3 \cdot 3 \cdot 1 \cdot 4 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$

7 6 5 4 3

$$\cdot 4 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 4 \cdot 3 \cdot 3 \cdot 3$$

2

~~1 2 3~~ ~~1 3 2~~

0 0 0 1 3 2

1 1

1 3 2

1 1 4

$\boxed{\begin{matrix} 1 & 1 & 3 \\ 1 & 2 & 2 \end{matrix}}$

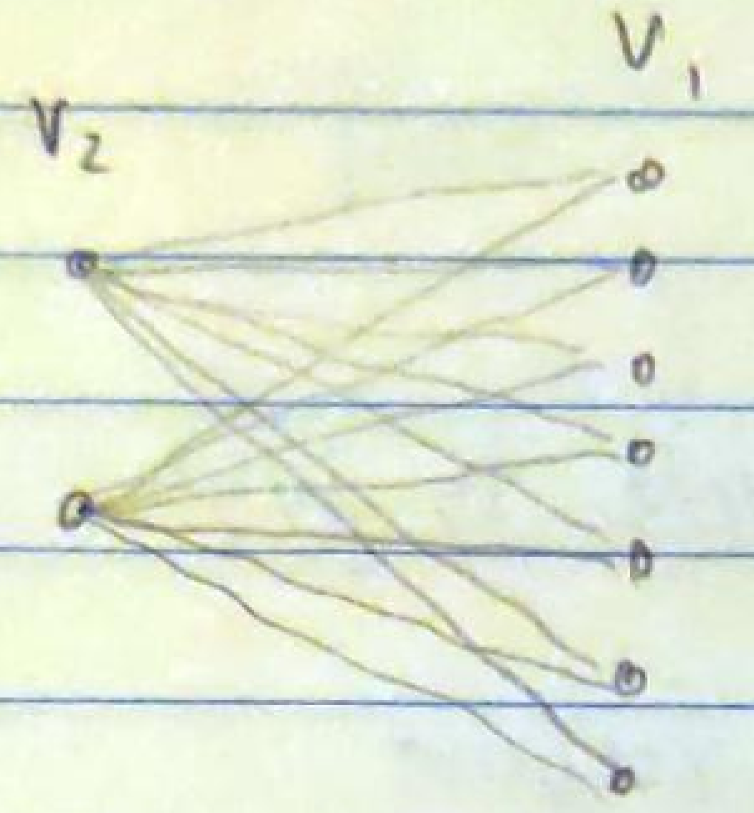
First we choose $C(10,7)$ which from part a.) is 120. Then by multiplication principle we mutiply bey the number of valid distributions which is 7^4 b/c each hobbit has 7 choices of books to take. Thus we have

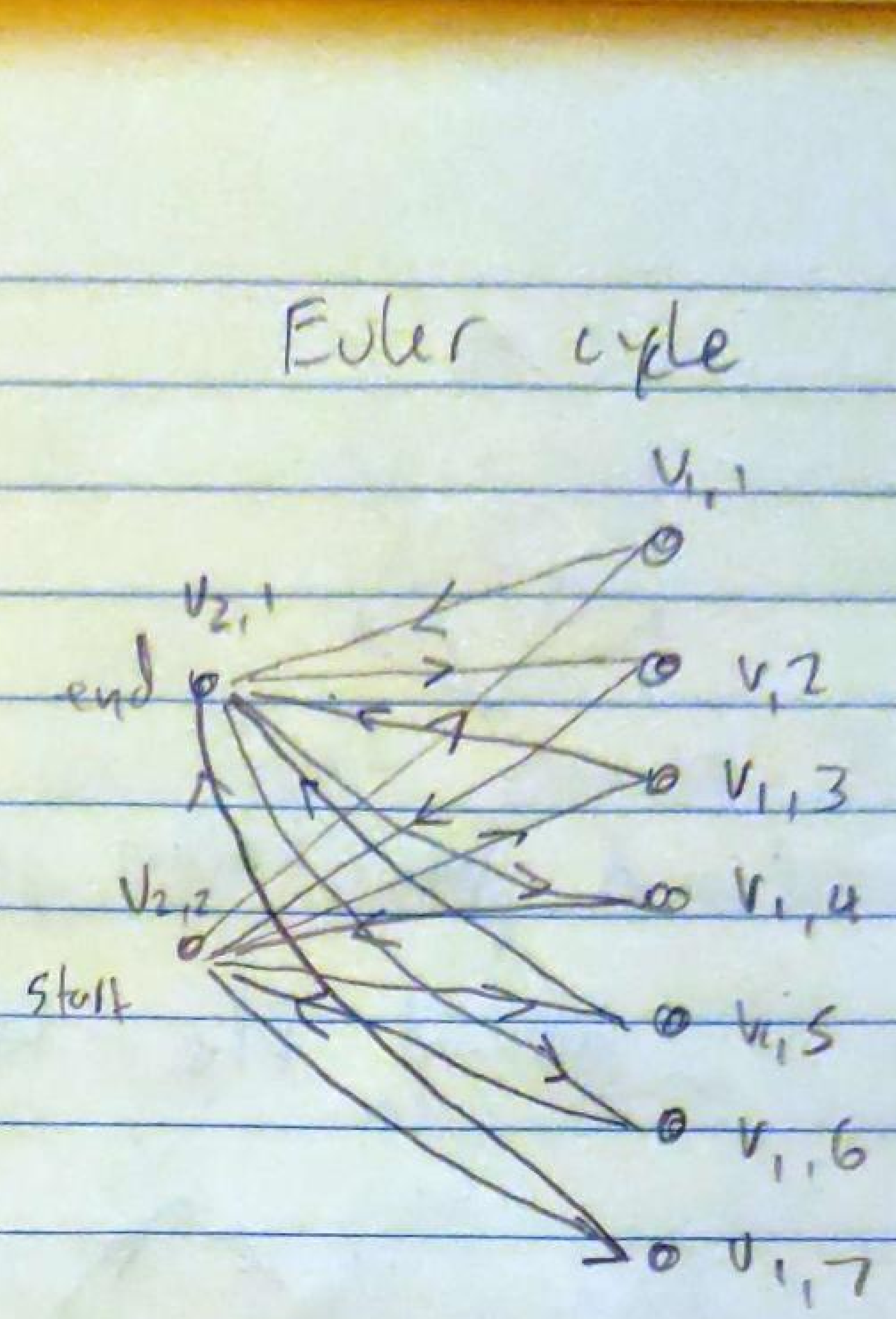
$\boxed{7^4 \cdot 120 = 288120}$

Mod 2 - q 2b

Euler cycle

a.)





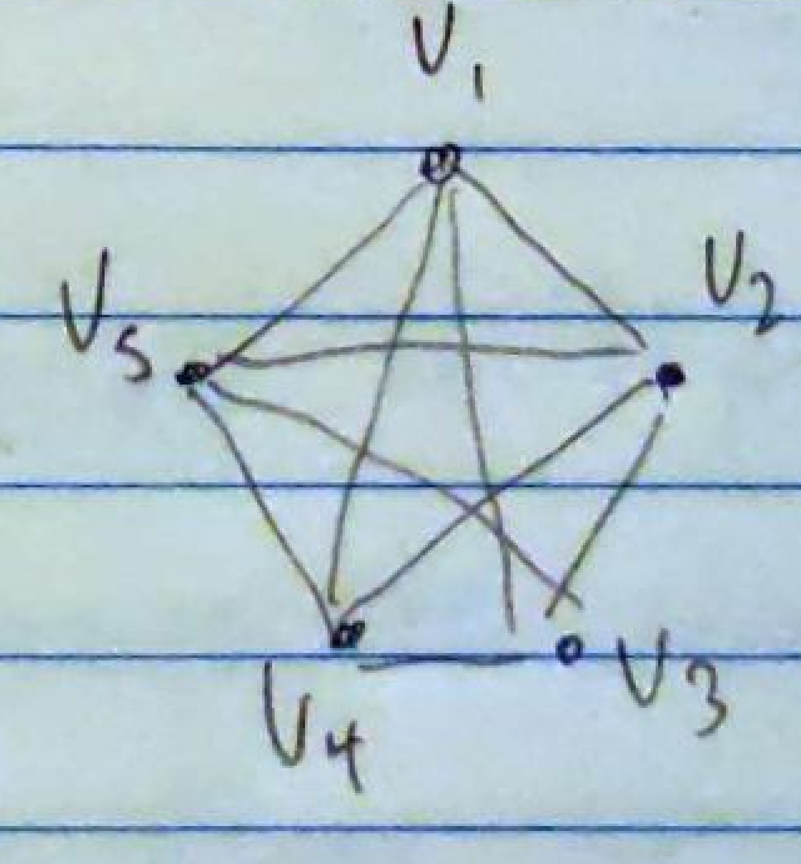
can also be represented as

$V_{2,2}$ $V_{1,1}$ $V_{2,1}$ $V_{1,2}$ $V_{2,2}$ $V_{1,3}$

$V_{2,1}$ $V_{1,4}$ $V_{2,2}$ $V_{1,5}$ $V_{2,1}$ $V_{1,6}$ $V_{2,2}$ $V_{1,7}$

$V_{2,1}$

b.)



The Euler cycle
can be represented
as drawing a star then tracing a pentagon

or V_5 V_2 V_4 V_1 V_3 V_5
V_1 V_2 V_3 V_4 V_5

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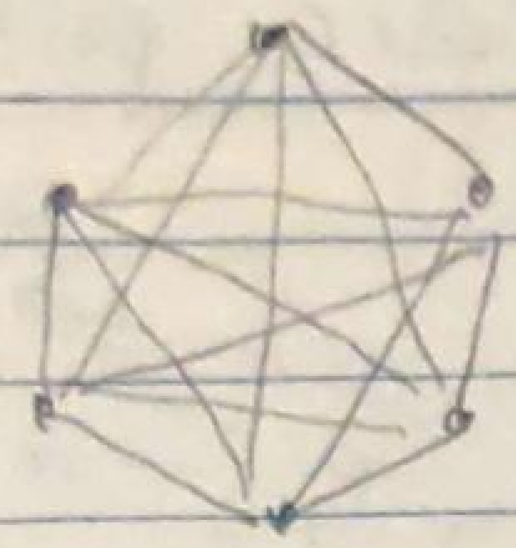
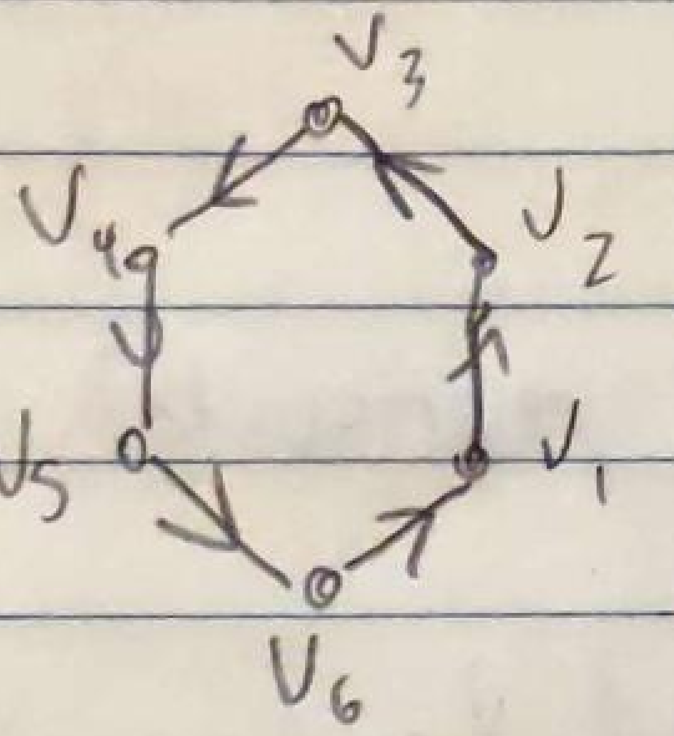
c.)



A hamiltonian cycle is just going around the Hexagon



or $V_1\ V_2\ V_3\ V_4\ V_5\ V_6\ V_1 \ldots$

Mid 2 - q3b

can be rewritten as

$$a_n = (f+1)a_{n-1} - f a_{n-2}, \quad n \geq 2$$

with $a_0 = 1$, $a_1 = 2 - f$

$$r^2 - (f+1)r + f$$

$$(r-f)(r-1) = 0$$

$$r = f, 1$$

$$a(n) = C_1 f^n + C_2$$

$$a(0) = 1 = C_1 + C_2$$

$$a(1) = 2-f = C_1 f + C_2$$

$$C_1 = 1 - C_2$$

$$(1 - C_2) f + C_2 = 2 - f$$

$$2f - C_2 f + C_2 = 2$$

$$2f - 2 = C_2 f - C_2 \quad \rightarrow$$

$$\cancel{2(f-1)} = c_2\cancel{(f-1)}$$

$$c_2 = 2$$

$$c_1 + c_2 = 1$$

$$c_1 + 2 = 1$$

$$c_1 = -1$$

thus the equation is

$$\boxed{a(n) = -(f^n) + 2}$$

Mid 2 - q4d

Part 1.

given that there are 5 answers per question, and 3 questions, there are 5.5.5 possible unique solutions to the quiz.

$$5 \cdot 5 \cdot 5 = 125 \text{ unique solutions.}$$

Since the professor has 130 students, there is no way for each of them to all have unique solutions, at least 10 students will have the same answer as someone else by the pigeonhole principle. In this case, there are 125 pigeonholes but 130 pigeons.

Part 2.

if we split the numbers up into 10 distinct pigeonholes from

50 - 54 , 55 - 59 , . . . , 95 - 99

and we begin placing houses in these pigeonholes, we can place 40 houses into 10 pigeonholes so that each pigeonhole has 4 houses, but our extra 1 house will have to be placed into a pigeonhole with 4 houses already thus bringing it to 5 houses which would mean all houses in that pigeonhole are consecutive, and since there are 5 houses, there would have to be 5 consecutively numbered houses.

Part 2

o o o o o o o o o

5^9 total options

by mutiplication principle there are

$$\underbrace{1\cdot 1\cdot 1\cdot 1\cdot 1}_{\text{must order 1 of each kind}}\cdot\underbrace{5\cdot 5\cdot 5\cdot 5}_{\text{then they can order whatever they want.}}$$

thus we have 5^4 ways =

$\boxed{\text{625 ways to order.}}$