Q1A

Let $P(n)$ be the statement "$2^{n+1} \geq 2n+2$" for $n > 0$

base: $P(1)$ or $n = 1$

$2^2 \geq 2+2 \rightarrow 4 \geq 4$ ✓ true

inductive step:

Assume $P(n)$ is true, now we must prove $P(n+1)$ is true.

from inductive hypothesis we know

$$2n+2 \leq 2^{n+1}$$

this implies

$2n \leq 2^{n+1}$ which also implies that

$4 \leq 2^{n+1}$ given $n > 0$

thus

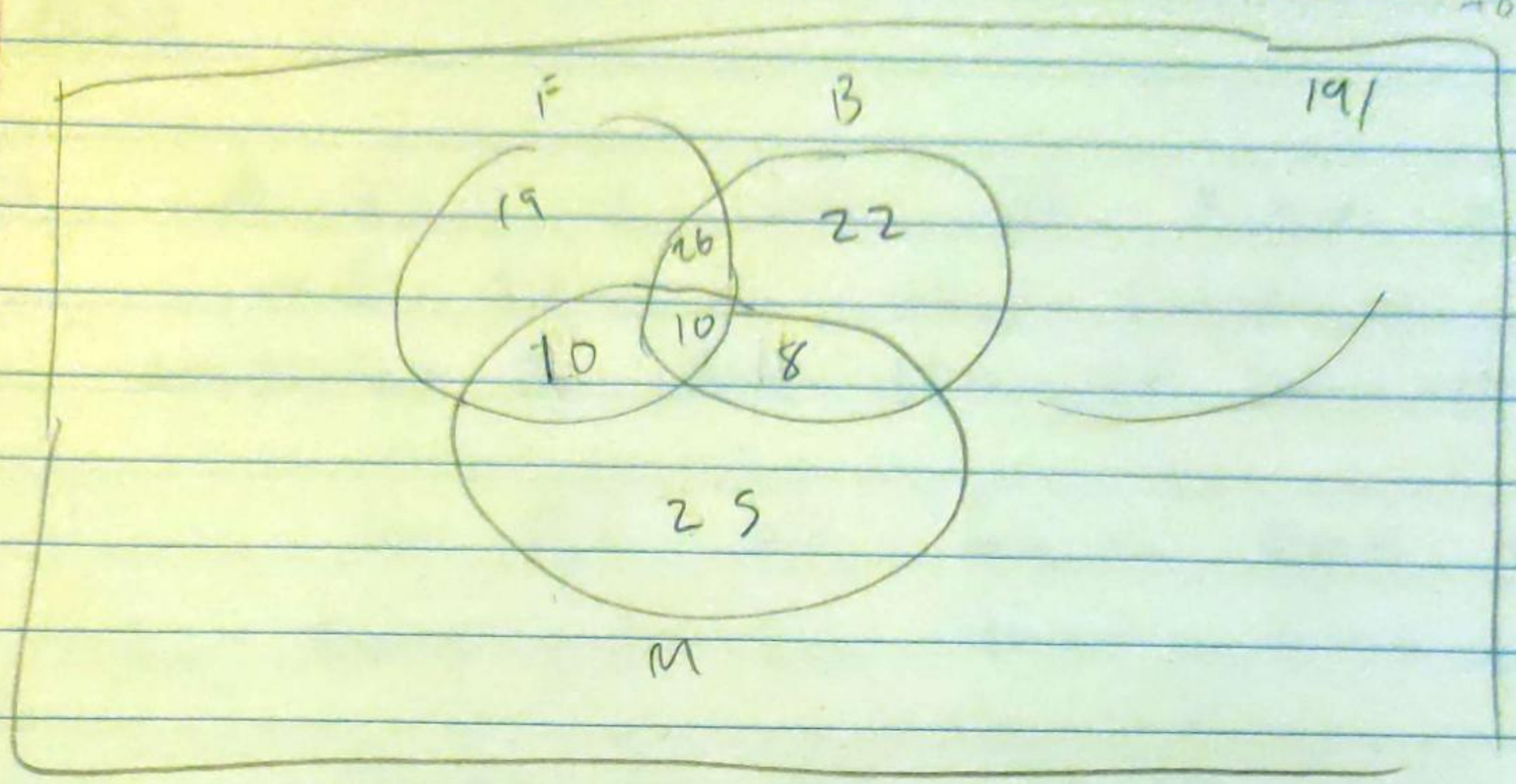
$$2n+4 \leq 2^{n+1} + 2^{n+1}$$

$$2n+4 \leq 2 \cdot 2^{n+1}$$

$$2n+4 \leq 2^{n+2}$$

$$\boxed{2(n+1)+2 \leq 2^{(n+1)+1}}$$

Q2C



1. 26 are taking french & business but not music

2. 19 are not taking business or music or both.

Q3A

1. λ is in L b/c let $\lambda = \alpha$ $\lambda = \beta$
$\alpha\beta = \lambda\lambda = \lambda$ thus satisfying condition 2 From this we know that

by Rule 1 $ba \in L$ b/c $ba = ba\alpha$ $\alpha = \lambda$

thus $bbaa \in L$ b/c $bbaa = b\alpha a$ $\alpha = ba$

thus $abbaab \in L$ b/c $abbaab = a\alpha b$ $\alpha = bbaa$

2. no because the null string is the only starting point upon which to build strings and thus strings will only be built with an even number of characters due to rule 1. Thus, since

abaab has an odd number of characters, it can never be constructed b/c

rule 1 only adds an even number of characters as does rule 2, since it adds together 2 existing strings (which will be even) thus creating more even strings making an odd character count string such as abaab impossible.

Q4D

R, Equivalence relation b/c is

- reflexive: if A owns n cars, the A also owns n cars thus itself is in the relation.
- symmetric: if A owns n cars and B owns n cars, then also if B owns n cars A owns n cars. Thus (A,B) is in the relation as is (B,A) since the number of cars they own is the same, all n cars.
- transitive: if A owns n cars and B owns n cars, then (A,B) is in the relation since they own the same number of cars. if C also owns n cars, then (B,C) own the same number of cars and is also in the relation. By the same logic (A,C) is also in the relation since $n_A = n_B = n_C$ thus showing transitivity.

#2. Not an equivalence relation since not symmetric:

if x is younger than y, then (x,y) is in the relation. But if x is younger than y, y is not younger than x, thus (y,x) is not in the relation which shows that it is not symmetric and thus not an equivalence relation.

Q5D

The shirt

Riverdale

On the way to morder, there are

$2 \cdot 3 \cdot 2 \cdot 2 \quad 24$

$+ \quad 1 \cdot 1 \qquad +6$ > 30 routes to morder

30 routes to, 29 routes back = $30 \cdot 29$

= 870 total routes.