20S-MATH61-2 Midterm 1

GEORGE OWEN

TOTAL POINTS

40 / 50

QUESTION 1

1 Question 1 4 / 10

- **- 0 pts** Correct, or mostly correct.
- ✓ **- 4 pts Major algebraic/conceptual error.**
- **- 2 pts** Misleading notation, circular reasoning, or wrong proof direction (necessary instead of sufficient).
- ✓ **- 2 pts Conceptual error.**
- **- 1 pts** Minor conceptual error/typo, or unproven claim.
- **- 6 pts** Not clear what is happening, or incorrect proof.

1 No. This is a major mistake. For example, when k = 0, the left hand side is 16, whereas the right hand side is 4 + 1 = 5.

QUESTION 2

Question 2 10 pts

2.1 Part i. 4 / 5

- **- 0 pts** Correct
- ✓ **- 1 pts Minor mistake.**
- **- 2.5 pts** Incorrect.

2 Should be 26.

2.2 Part ii. 5 / 5

- ✓ **- 0 pts Correct**
- **- 1 pts** Minor mistake.
- **- 2 pts** Conceptual mistake.
- **- 2.5 pts** Incorrect.
- **- 5 pts** No submission.

QUESTION 3

Question 3 10 pts

3.1 Part i. 6 / 6

- ✓ **- 0 pts Correct**
- **- 4 pts** Incorrect assumption/proof (see note).
- **- 2 pts** Incorrect.
- **- 1 pts** Minor mistakes/misuse of notation.
- **- 5 pts** Vague/ambiguous statement(s).
- **- 6 pts** No submission.

3.2 Part ii. 4 / 4

- ✓ **- 0 pts Correct**
- **- 2 pts** Incorrect assumption/explanation/proof.
- **- 4 pts** No submission.
- **- 2 pts** False/Unproven/unexplained claim.
- **- 0.5 pts** Unproven/unexplained believable claim.
- **- 1 pts** Correct idea, but incomplete explanantion/proof.
- **- 3.5 pts** Vague/ambiguous statement(s).

QUESTION 4

Question 4 10 pts

4.1 Part i. 5 / 5

- **- 0 pts** Correct
- ✓ **- 0 pts Correct, but see note.**
- **- 1 pts** Poor/Inaccurate argumentation (see note).
- **- 4 pts** Not a proof, or incorrect argumentation (see note).
- **- 2 pts** Missing argument(s).

3 Scratch this.

By writing "By the INSERT NAME HERE property", you are giving the impression that you are using said property.

You are not using it, instead you are proving it.

4

Nicely done. You are abusing notation; that is, using the symbols | | outside of their usual context. When doing so, a note like this helps the reader understand the new context.

4.2 Part ii. 5 / 5

- **- 0 pts** Correct
- ✓ **- 0 pts Correct, but see note.**
- **- 2 pts** Incorrect. See note.
- **- 0.5 pts** Little to no arguments provided.
- **- 0.5 pts** Correct idea, but poor argumentation.
- **- 4 pts** Not a proof, or incorrect argumentation (see note).
- **- 1 pts** Incorrect claim (see note).
- **- 2 pts** Missing argument(s).

5 We have defined "symmetric" and "anti-symmetric". Here, by "asymmetric" you clearly mean not symmetric... but in math, sometimes terminology is not intuitive (e.g., not open, does not mean closed, sets could be both open and closed). Next time, use terms that avoid ambiguity.

QUESTION 5

5 Question 5 7 / 10

- **- 0 pts** Correct
- **- 2 pts** Incorrect/Incomplete answer (see note).
- **- 3 pts** Partial, or incorrect answer (see note).
- ✓ **- 4 pts Incorrect, but partial credit awarded.**
- **- 0.5 pts** Incorrect answer, but [mostly] correct procedure.
- **- 2 pts** Incorrect partial answer (see note).
- **- 0 pts** Attempted a different problem.
- **- 10 pts** Missing submission.
- **- 1 pts** Minor mistake.

+ 1 Point adjustment

Some correct use of counting principles.

6 It appears you wanted all legs to be different. The answer is

For shortcut:

* "there" paths = 3*2*2 =12
* "back" paths = (3*1+1)*1*2 = 8

For no shortcut:

* "there" paths = 3*2*1*3 = 18
* "back" paths = 2*1*2 = 4

Total = 12*8 + 18*4 = 168

Midterm 1

George Owen
Math 61
4/27/20

(Q1C) Prove: $4^{n+1} \geq 4n+4$ for $n>0$

Proof:

Base case of $n=1$:

$$4^{1+1} = 16$$
$$4(1)+4 = 8$$
$$\checkmark\ 16 \geq 8$$

$n \in \mathbb{N}$? (not specified)

$n=1.5$

$4^{2.5} = 32$

$4(1.5)+4 = 10$

$\checkmark\ 32 \geq 10$

Inductive Hypothesis:

Let's assume that, like the base case,

$$4^{k+1} \geq 4k+4 \text{ for some } k$$

Now, let's prove the same relation holds for $k+1$

$$4^{(k+1)+1} = 4^{k+1} + 4^k$$

$$4(k+1)+4 = (4k+4)+4$$

Since $4^{k+1} \geq 4k+4$ by our inductive hypothesis, it holds that

$$(4^{k+1}) + 4^k \geq (4k+4)+4$$

for all $k>0$.

Thus, we have shown that

$$4^{n+1} \geq 4n+4 \text{ for } n>0$$

by induction.

1 Question 1 **4 / 10**

- **- 0 pts** Correct, or mostly correct.
- ✓ **- 4 pts** **Major algebraic/conceptual error.**
- **- 2 pts** Misleading notation, circular reasoning, or wrong proof direction (necessary instead of sufficient).
- ✓ **- 2 pts** **Conceptual error.**
- **- 1 pts** Minor conceptual error/typo, or unproven claim.
- **- 6 pts** Not clear what is happening, or incorrect proof.

(1) No. This is a major mistake. For example, when k = 0, the left hand side is 16, whereas the right hand side is 4 + 1 = 5.

Q20 take 2

$10 = |F \cap B \cap M|$

$36 = |F \cap B|$

$20 = |F \cap M|$

$18 = |B \cap M|$

$65 = |F|$

$76 = |B|$

$63 = |M|$

$191 = |S|$

F∪M∪B

$F \cup M \cup B = 76 + 63 + 65 - (36 + 20 + 18) + 10$

$76 + 63 + 65 - 74 + 10 = 140$

$= 140$ students in French, Music, or Business

51 in none

(1) $|F \cap B| - |F \cap M| - |B \cap M| + |F \cap B \cap M| =$

$36 - 20 - 18 + 10 =$ **8 students taking Business & French, but not music**

(2) # in just music + # in none

$|M| - |F \cap M| - |B \cap M| + |F \cap B \cap M| + 51$

$63 - 20 - 18 + 10 + 51 =$ **86 students not taking French or Business**

2.1 Part i. **4 / 5**

- **- 0 pts** Correct
- ✓ **- 1 pts Minor mistake.**
- **- 2.5 pts** Incorrect.

2 Should be 26.

Q20 Take 2

$10 = |F \cap B \cap M|$

$36 = |F \cap B|$

$20 = |F \cap M|$

$18 = |B \cap M|$

$65 = |F|$

$76 = |B|$

$63 = |M|$

$191 = |S|$

$F \cup M \cup B = 76 + 63 + 65 - (36 + 20 + 18) + 10$

$76 + 63 + 65 - 74 + 10 = 140$

$= 140$ students in French, Music, or Business

51 in none

(1) $|F \cap B| - |F \cap M| - |B \cap M| + |F \cap B \cap M| =$

$36 - 20 - 18 + 10 =$ 8 students taking Business & French, but not music

(2) # in just music + # in none

$|M| - |F \cap M| - |B \cap M| + |F \cap B \cap M| + 51$

$63 - 20 - 18 + 10 + 51 =$ 86 students not taking French or Business

2.2 Part ii. 5 / 5

- ✓ **- 0 pts** Correct
- **- 1 pts** Minor mistake.
- **- 2 pts** Conceptual mistake.
- **- 2.5 pts** Incorrect.
- **- 5 pts** No submission.

Q3D

{} ∈ L

$\alpha \in L \Rightarrow a\alpha b \in L,\ b\alpha a \in L$

$\alpha \in L\ \&\ \beta \in L, \Rightarrow \alpha\beta \in L$

(1) Since {} ∈ L, we know that

ab ∈ L and ba ∈ L, wher {} is α

abbaab

Using "ba" as α now, we know that

bbaa ∈ L,

Now consider bbaa as α, and we see that

abbaab ∈ L, all by our first hypothesis

(2) "aabab" is NOT in L.

We can see that all of the strings used in part 1 contain a number of elements corresponding to 2n, where n≥0 (and, in the case of 1, corresponds to the number of operations applied to transform the empty string to it.)

That is, all the strings in this set contain an even number of elements.

Both operations we have involve adding an even number of elements, or combining 2 existing strings. Since we are not given any strings w/ an odd number of elements, and the set of evens is closed under addition, we conclude that odd strings ∉ L,

⇒ aabab ∉ L

□

3.1 Part i. **6 / 6**

✓ **- 0 pts** **Correct**

- **- 4 pts** Incorrect assumption/proof (see note).
- **- 2 pts** Incorrect.
- **- 1 pts** Minor mistakes/misuse of notation.
- **- 5 pts** Vague/ambiguous statement(s).
- **- 6 pts** No submission.

Q3D

$\{\} \in L$

$\alpha \in L \Rightarrow a\alpha b \in L, b\alpha a \in L$

$\alpha \in L$ & $\beta \in L, \Rightarrow \alpha\beta \in L$

1. Since $\{\} \in L$, we know that

ab $\in L$ and ba $\in L$, wher {} is α

abbaab

Using "ba" ~~ab~~ as α now, we know that

bbaa $\in L$, ~~abba~~

Now consider bbaa as α, and we see that

abbaab $\in L$, all by our first hypothesis

2. "aabab" is NOT in L.

We can see that all of the strings used in part 1 contain a number of elements corresponding to 2n, where $n \geq 0$ (and, in the case of 1, corresponds to the number of operations applied to transform the empty string to it.)

That is, all the strings in this set contain an even number of elements.

Both operations we have involve adding an even number of elements, or combining 2 existing strings. Since we are not given any strings w/ an odd number of elements, and the set of evens is closed under addition, we conclude that odd strings $\notin L$,

$\Rightarrow$ aabab $\notin L$ □

3.2 Part ii. **4 / 4**

✓ **- 0 pts** **Correct**

- 2 pts Incorrect assumption/explanation/proof.

- 4 pts No submission.

- 2 pts False/Unproven/unexplained claim.

- 0.5 pts Unproven/unexplained believable claim.

- 1 pts Correct idea, but incomplete explanantion/proof.

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(Q4D) Equivalence Relations are reflexive, symmetric, and transitive

$R_1 = \{(x,y) \mid x \text{ \& } y \text{ have the same number of cars}\}$

Suppose person x owns m cars. By the reflexive property, x owns the same amount of cars as himself, so

$(x,x) \in R_1$ and R_1 is reflexive

Suppose person y also owns m cars. Then, $(x,y) \in R_1$.

Since y owns m cars and x owns m cars, ~~that~~

$(y,x) \in R_1$ by the symmetric property of algebra,

and R_1 is symmetric.

Finally, suppose person z also owns m cars. Since

$|x| = |y| = |z| = m$, we know that

(number of cars x owns → $|x|$)

$|x| = |y| \Rightarrow (x,y) \in R_1$

$|y| = |z| \Rightarrow (y,z) \in R_1$

$|x| = |z| \Rightarrow (x,z) \in R_1$,

which means R_1 is transitive

Since R_1 is reflexive, symmetric, and transitive, it is an equivalence relation.

$R_2 = \{(x,y) \mid x \text{ is younger than } y\}$

Suppose person m is 15 and person k is 30.

with $m = x$ and $k = y$, $(m,k) \in R_2$

however, with $k = x$ / $m = y$, $(k,m) \notin R_2$

Meaning R_2 is asymmetric and therefore not an equivalence relation.

(Q5C)

S → R
3 2

cannot ref
use

3·2·2

there

4.1 Part i. **5 / 5**

- **- 0 pts** Correct
- ✓ **- 0 pts Correct, but see note.**
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3 Scratch this.

By writing "By the INSERT NAME HERE property", you are giving the impression that you are using said property.

You are not using it, instead you are proving it.

4 Nicely done. You are abusing notation; that is, using the symbols | | outside of their usual context. When doing so, a note like this helps the reader understand the new context.

(Q4D) Equivalence Relations are reflexive, symmetric, and transitive

$R_1 = \{(x,y) \mid x \,\&\, y \text{ have the same number of cars}\}$

Suppose person x owns m cars. By the reflexive property, x owns the same amount of cars as himself, so $(x,x) \in R_1$ and R_1 is reflexive

Suppose person y also owns m cars. Then, $(x,y) \in R_1$. Since y owns m cars and x owns m cars, ~~then~~ $(y,x) \in R_1$ by the symmetric property of algebra, and R_1 is symmetric.

Finally, suppose person z also owns m cars. Since $|x| = |y| = |z| = m$, we know that

(number of cars x owns → $|x|$)

$$|x| = |y| \Rightarrow (x,y) \in R_1$$
$$|y| = |z| \Rightarrow (y,z) \in R_1$$
$$|x| = |z| \Rightarrow (x,z) \in R_1,$$

which means R_1 is transitive

Since R_1 is reflexive, symmetric, and transitive, it is an equivalence relation.

$R_2 = \{(x,y) \mid x \text{ is younger than } y\}$

Suppose person m is 15 and person k is 30, with m=x and k=y, $(m,k) \in R_2$ however, with k=x / m=y, $(k,m) \notin R_2$

Meaning R_2 is asymm[illegible] and therefore not an equivalence relation.

(Q5C)

5 → R
3 2

~~3·2·1·3·…~~

cannot ret…
use…

3·2·2…
there…

4.2 Part ii. 5 / 5

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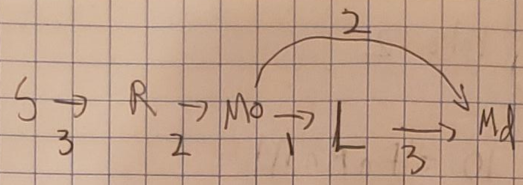
Q5C



~~3·2·1·3 + 3·2·2~~ = going there

cannot return from Lothlórien to Moira, so we can't use that path or any from Lothlórien to Mordor

$\underbrace{3\cdot 2\cdot 2}_{\text{there}} \cdot \underbrace{1\cdot 1\cdot 2}_{\text{back}} =$ **24 ways to make a round trip**

5 Question 5 7 / 10

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