

Name: _____

UID: _____

Instructions:

- You are allowed a one page cheat sheet (front and back), but no other notes or resources.
- Show your work. Full points are only given for correct answers *with* adequate justification. A correct final answer with missing or substantially incorrect justification will not merit full points on a problem.

Problem 1. Prove by induction that

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

for all $n \geq 1$.

Problem 2. Let the function $f : \mathcal{P}(\{1, 2, 3, 4\}) \rightarrow \mathcal{P}(\{1, 2, 3\})$ be defined so that

$$f(X) = X \cap \{1, 2, 3\}$$

for all $X \in \mathcal{P}(\{1, 2, 3, 4\})$ (Recall that, for a set Y , $\mathcal{P}(Y)$ denotes the power set of Y).

1. Is f injective?
2. Is f surjective?

Justify your answers.

Problem 3. Recall that if k and n are integers, we say that k is a multiple of n if there is some integer m such that $k = nm$. Suppose $X = \{1, 2, \dots, 36\}$. Define sets X_2 and X_3 by

$$X_2 = \{k \in X : k \text{ is a multiple of } 2\}$$

$$X_3 = \{k \in X : k \text{ is a multiple of } 3\}.$$

1. What is $|X_2|$?
2. What is $|X_2 - X_3|$?
3. What is $|X_2 \cup X_3|$?

Justify your answers.

Problem 4. The *symmetric difference* of two sets X and Y , denoted $X\Delta Y$, is the set of elements that are either in X or in Y but not in both; formally, we define $X\Delta Y$ by

$$X\Delta Y = (X \cup Y) - (X \cap Y).$$

1. What is $\{1, 2, 3, 4, 5\} \Delta \{2, 4, 6, 8, 10\}$?
2. Suppose R and S are relations on a non-empty set Z . Assume R is reflexive. Show that $R\Delta S$ is reflexive if and only if S is irreflexive.

