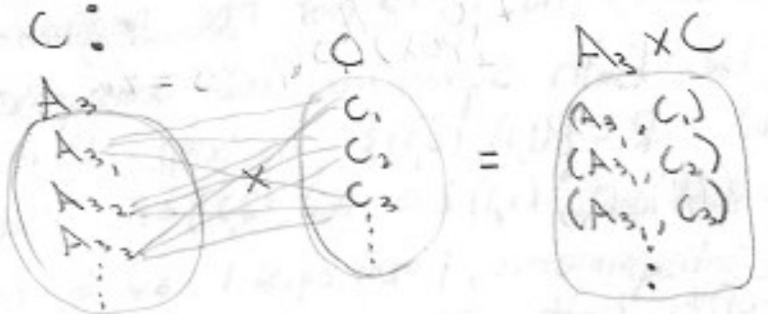
① Base

$\boxed{n=2}$ $\left(\bigcup_{i=1}^{2} A_i\right) \times C = \bigcup_{i=1}^{2} (A_i \times C)$ $\quad (A_1 \cup A_2) \times C = (A_1 \times C) \cup (A_2 \times C)$

- $A_1 \cup A_2$ produces a unique set (lets call it A_3). The cartesian product of $A_3 \times C$ has each element in A_3 matched with every element in C:



- $(A_1 \times C)$ has each element in A_1 match with every element in C. Equally $A_2 \times C$ has each element in A_2 mached with every element in C. When there is a union between these sets, since both A_1 and A_2 took the cartesian product with C (the same set), The x values of $(A_1 \times C) \cup (A_2 \times C)$ will be equivalent to $A_1 \cup A_2$. $A_1 \cup A_2 = A_3$. Since the x values $= A_3$, the cartesian product of this set with C will be equivalent to $A_3 \times C$.

Inductive step Assuming $\left(\bigcup_{i=1}^{n} A_i\right) \times C = \bigcup_{i=1}^{n} (A_i \times C)$, then

$\boxed{n=n+1}$ $\left(\bigcup_{i=1}^{n+1} A_i\right) \times C = \bigcup_{i=1}^{n+1} (A_i \times C)$.

Since the cartesian product with C is a distributive property, and unions can be regrouped $(A \cup B) \cup C = A \cup (B \cup C)$, the result will always be the same Cartesian product.

$(A_1 \cup A_2 \cup A_3 \ldots \cup A_n \cup A_{n+1}) \times C = (A_1 \cup A_2 \ldots \cup A_n) \times C \cup (A_{n+1} \times C)$

end up with the same x values (A_3)

taking the same cartesenian product $(\times C)$

$$(A_1 \times C) \cup (A_2 \times C) \ldots \cup (A_n \times C) \cup (A_{n+1} \times C) = (A_1 \times C) \cup (A_2 \times C) \ldots \cup (A_{n+1} \times C$$

$$\left[\left(\left(\bigcup_{i=1}^{n} A_i\right) \times C\right) = \bigcup_{i=1}^{n} (A_i \times C)\right] \cup (A_{n+1} \times C)$$

2. "R is anti-symmetric" is not the negation of "R is symmetric".

a) To be symmetric, a relation that includes (x,y) must include (y,x) for all x,y values in X. The negation for this property would be $(x,y) \in R$ and $(y,x) \notin R$, but this is not equivalent to the antisymmetric property. The antisymmetric property states that if both (x,y) and $(y,x) \in R$, then $x=y$. We can be sure that it is not the negation because a relation can be both symmetric and antisymmetric.

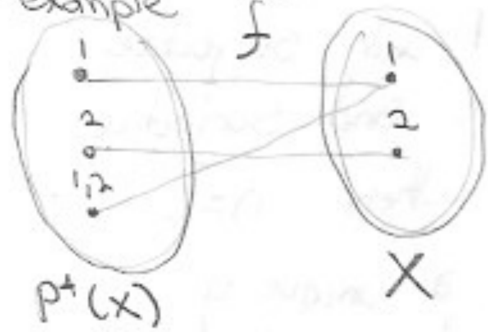
If $x,y = \{1,2\}$ and $R = \{(1,1),(2,2)\}$ $(x=y)$. R is symmetric because for all $x,y \in X$ $(x,y) \in R$ and $(y,x) \in R$; $(1,1) \in R$ and $(2,2) \in R$. Since $(1,1),(2,2) \in$ for this to be antisymmetric, 1 must equal 1 and 2 must equal 2. This relation fufills both properties.

b) "R is antireflexive" is not the negation of "R is reflexive". To be reflexive, for all $x \in X$, (x,x) must be in relation. To be antireflexive for all $x \in X$, (x,x) must not be in relation. $\neg(A \wedge B) \neq A \wedge \neg B$. We can be sure this is not the negation because a Relation can be not reflexive and not irreflexive.

If $x,y = \{2,4,6\}$ and $R = \{(2,2)\}$

- R is not reflexive because $(4,4),(6,6) \notin R$
- R is not antireflexive because $(2,2) \in R$

(3) example



$$\boxed{\begin{gathered} n>1 \\ X=\{x,y\ldots\} \\ P^*(X)=\{\{x\},\{y\},\{x,y\}\ldots \end{gathered}}$$

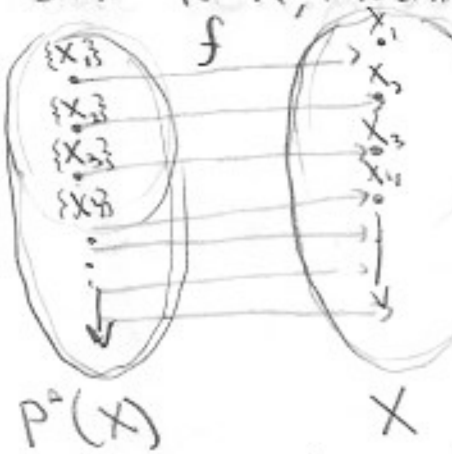
- Say $x,y\in X$, $x<y$ and $n>1$. $\{x,y\}\in P^*(X)$, and $\{x\}\in P^*(X)$. $f(\{x,y\})=x$ and $f(\{x\})=x$. Thus $f(\{x,y\})=f(\{x\})$. But $\{x,y\}\neq\{x\}$, thus for $n>1$, each input does not have a unique output.

- However, say $n=1$ and $x\in X$ so X consists of only x. the only set in $P^*(X)$ will be $\{x\}$.

$$\boxed{\begin{gathered} n=1 \\ X=\{x\} \\ P^*(X)=(\{x\}) \end{gathered}}$$

$f(\{x\})=x$ is the only relation, so there is one input with one unique output.
Thus f is injective when $n=1$.

- A $P^*(X)$ is a power set of X (minus the null) meaning when n is a pos, natural number and some $x\in X$, then $\{x\}\in P^*(X)$. Since f sends each nonempty subset of X to its least element, $\{x\}$ will always be sent to x, meaning the range $=X$.



Since every $x\in X$ has $\{x\}\in P^*(X)$, the codomain = range and f is surjective for all n.

Since f is Injective when $n=1$, and Surjective for all n, f is both Injective and Surjective when $n=1$. Thus f is bijective when $n=1$.

- each x has a unique y
- for every $y \in Y$, there exists $x \in X$, such that $(f(x) = y)$.

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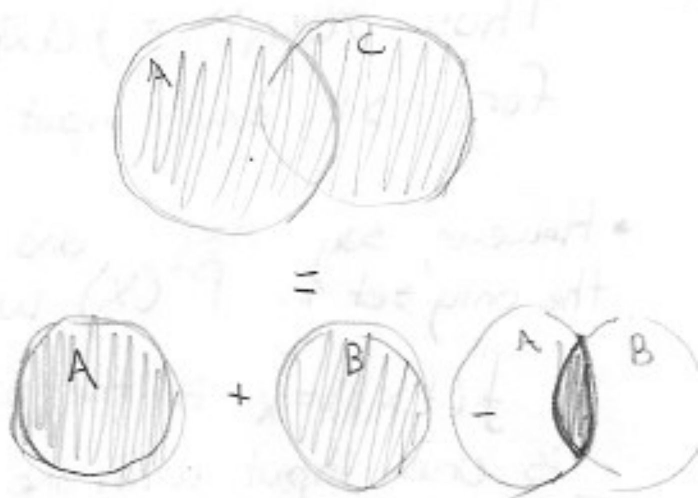
(4) A = Averie is first C = Charlie is last

$(A \cup C) \cup (A \cap C)$

$\hookrightarrow (A \cup C \cup A) \cap (A \cup C \cup C)$

$\hookrightarrow (A \cup C) \cap (A \cup C)$

$\hookrightarrow A \cup C$



A| . . . . . . 16! . . . . . .

when A is set to the front

. . . . . . 16! . . . . . . |C

when C is set to the end

A| . . . . 15! . . . . |C

when A and C are set.

- When A is first, you're actually only ordering 16 student.
- When C is last, ording 16 students
- When A is first, C last, ordering 15 students

$|A \cup C| = |A| + |C| - |A \cap B|$

$= \boxed{16! + 16! - 15!}$