Math 61 – Fall ’21 – Midterm 1 Practice

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Not due. Some of the problems may appear on the next homework.

Exercises from the book. NOTE: The book contains “Review exercises” and “Exercises”. Unless otherwise specified, the exercise numbers on the homework refer to the “Exercises”, **not** to the “Review exercises”.

Section 3.2. 27, 80, 82, 85.
Hint for 85: Calculate the difference between two consecutive terms. Then use problem 1 below.

Section 3.3. 25, 32, 35, 41, 42, 58, 62.

Section 3.4. 6, 9, 29, 41, 44. Optional: 67, 68, 69.

Section 6.1. 23, 24, 28, 31, 93, 98.

Problem 1. Suppose $s_n, n \geq 1$ is a sequence such that for all $i \geq 1$: $s_i < s_{i+1}$. Show that s is increasing.
Hint: Fix $i \geq 1$ and show by induction on j that for each j with $i < j$, we have $s_i < s_j$.

Remark: Of course, the same proof shows that if we assume instead that $s_i > s_{i+1}$, then s is decreasing.

Problem 2. Let X and Y be sets, and denote by Y^X the set of all functions from X to Y.

(a) Find $\left|Y^X\right|$. (In other words, how many functions are there from X to Y?)

(b) Given $A \subset X$, consider the function $f_A : X \to \{0, 1\}$ defined by

$$f_A(x) = \begin{cases} 1, & \text{if } x \in A, \\ 0, & \text{if } x \notin A. \end{cases}$$

Show that the function $f : \mathcal{P}(X) \to \{0, 1\}^X : f(A) = f_A$ is a bijection.

(c) Explain briefly how it follows from (a) and (b) that $|\mathcal{P}(X)| = 2^{|X|}$.