211A-MATH61-1 Midterm 1

MENGAN WANG

TOTAL POINTS

97 / 100

QUESTION 1

Question 1 20 pts

1.1 Part (a) 10 / 10

✓ **- 0 pts Correct**

1 What do you mean by "their sizes"?

1.2 Part (b) 10 / 10

✓ **- 0 pts Correct**

QUESTION 2

2 Question 2 20 / 20

✓ **- 0 pts Correct**

QUESTION 3

Question 3 20 pts

3.1 Part (a) 10 / 10

✓ **- 0 pts Correct**

3.2 Part (b) 8 / 10

✓ **- 2 pts Proved a different & stronger result without acknowledging.**

2 x is given, not y. You cannot "take" y. You just proved

"there exists y s.t. for all x, x^2 + y^2 is not equal to 4"

(which implies what we want to prove, btw.)

QUESTION 4

4 Question 4 19 / 20

✓ **- 1 pts Issues with wording or presentation**

3 You only need to do 1

4 This what you are trying to prove, you can't just assume it. You mean something like "Assume $$A_n=n/(n+1)$$ for some $$n$$. We want to prove $$A_{n+1}=(n+1)/(n+2)$$"

QUESTION 5

Question 5 20 pts

5.1 Part (a) 10 / 10

✓ **- 0 pts Correct**

5.2 Part (b) 5 / 5

✓ **- 0 pts Correct**

5.3 Part (c) 5 / 5

✓ **- 0 pts Correct**

QUESTION 6

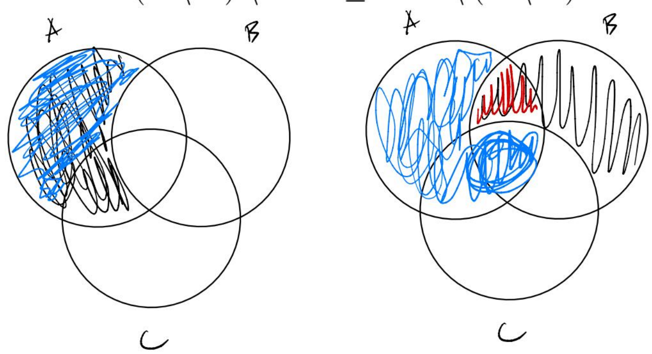
6 Academic Honesty Disclaimer 0 / 0

✓ **- 0 pts Correct**

1. (20 points) For all A, B and C, sets contained within a universal set $\mathcal{U}$, prove or give a counterexample to the following propositions.

   (a) (10 points)

$$(A \setminus B) \setminus C \quad \subseteq \quad A \setminus (B \setminus C).$$



I am going to prove (a)

Let A, B, C be sets within $\mathcal{U}$.

$(A\setminus B)\setminus C = (A \cap B^c) \setminus C$ by Set Difference Definition

$= A \cap (B^c \cap C^c)$ by Set Difference Definition and Associative Property of Intersection

$A \setminus (B \setminus C) = A \cap (B \cap C^c)^c$ by Set Difference Definition

$= A \cap (B^c \cup C)$ by De Morgan's law

$A \cap (B^c \cap C^c) \subseteq A \cap (B^c \cup C)$

Because both sides have A intersecting with something else, we can compare their sizes.

$B^c \cap C^c \subseteq B^c$. $B^c \subseteq B^c \cup C$

$\therefore$

$B^c \cap C^c \subseteq B^c \cup C$

Thus $(A\setminus B)\setminus C \subseteq A \setminus (B\setminus C)$ is true.

1.1 Part (a) 10 / 10

✓ **- 0 pts** **Correct**

1 What do you mean by "their sizes"?

(b) (10 points)

$$\mathcal{P}(A \setminus B) = \mathcal{P}(A) \setminus \mathcal{P}(B).$$

Here, $\mathcal{P}(X)$ means the power set of the set X.

I am going to give a counter example to disprove (b):

let $A = \{1, 2\}$ which satisfies $A \subseteq U$

let $B = \{1\}$ which satisfies $B \subseteq U$

Then $P(A \setminus B) = \{\emptyset, \{2\}\}$

and $P(A) \setminus P(B) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\} \setminus \{\emptyset, \{1\}\}$

$= \{\{2\}, \{1,2\}\}$

So $P(A \setminus B) \neq P(A) \setminus P(B)$ and we have shown at least one example that contradicts the statement.

1.2 Part (b) 10 / 10

✓ - 0 pts Correct

2. (20 points) Christopher Clavius, a German mathematician and astronomer who lived in the 16th century, proposes the following logical method:

   In order to ascertain the truth of a proposition P, one may attempt show that its negation would imply the proposition itself. Whenever this is the case, we could conclude P.

   Later, Bonaventura Cavalieri, an Italian mathematician, wanted to see if this Clavius' method is correct. Cavalieri expressed this method in logical notation as follows:

   $$(\neg P \implies P) \implies P.$$

   Using a truth table to justify your answer, find the truth value of this proposition. (Fun Fact: We owe the currently used Gregorian Calendar to Clavius' hard work.)

P	$\neg P$	$\neg P \to P$	$(\neg P \to P) \to P$
F	T	F	T
T	F	T	T

true

2 Question 2 **20 / 20**

✓ **- 0 pts** Correct

3. (20 points) Prove or disprove the following propositions.

(a) (10 points)

$$\forall x \in \mathbf{Z}, \quad x^2 - 6x + 5 \text{ is even} \implies x \text{ is odd.}$$

I am going to prove (3a) by showing proof by contrapositive:

Let us suppose x is not odd

Then for an integer y, $x = 2y$ so x is even.

Thus, $x^2 - 6x + 5 = (2y)^2 - 6(2y) + 5$

$= 4y^2 - 12y + 5$

$= 2(2y^2 - 6y + 4) + 1$

Let an integer $z = 2y^2 - 6y + 4$, so $x^2 - 6x + 5 = 2z + 1$

Therefore $x^2 - 6x + 5$ is odd when x is even, meaning that when $x^2 - 6x + 5$ is even x is odd. We have proved that if $x^2 - 6x + 5$ is even, then x is odd through proof of contrapositive.

3.1 Part (a) 10 / 10

✓ **- 0 pts** Correct

(b) (10 points)

$$\exists x \in \mathbb{R} \text{ s.t. } \forall y \in \mathbb{R}, x^2 + y^2 = 4.$$

I am going to disprove the proposition $\exists x \in \mathbb{R}$ s.t. $\forall y \in \mathbb{R}, x^2+y^2=4$ through proof by countcrexample

let $y = 3$, which satisfies $\forall y \in \mathbb{R}$ 2

Then $x^2 + (3)^2 = 4$

$x^2 + 9 = 4$

$x^2 = -5$

$x = \sqrt{-5} = 5i$

In order for the proposition to be true, $x = \sqrt{-5}$ which does not satisfy $x \in \mathbb{R}$.

so we have proved there is at least one case that contradicts $\exists x \in \mathbb{R}$ s.t. $\forall y \in \mathbb{R}, x^2+y^2=4$ and the proposition is false.

3.2 Part (b) **8 / 10**

✓ **- 2 pts Proved a different & stronger result without acknowledging.**

2 x is given, not y. You cannot "take" y. You just proved

"there exists y s.t. for all x, x^2 + y^2 is not equal to 4"

(which implies what we want to prove, btw.)

4. (20 points) Define A_n to be the following sum

$$A_n = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \ldots + \frac{1}{n\times(n+1)},$$

for each $n \in \{1, 2, 3, \ldots\}$. Using mathematical induction, prove that

$$A_n = \frac{n}{n+1}, \text{ for each } n \in \{1, 2, 3, \ldots\}$$

$A_1 = \frac{1}{1+1} = \frac{1}{2}$

Make sure to state the form of induction you are utilizing: "simple" or "strong".

Basic step:

If $n=1$, $A_1 = \frac{1}{1(1+1)} = \frac{1}{2}$

If $n=2$, $A_2 = \frac{1}{1(1+1)} + \frac{1}{2(3)} = \frac{2}{3}$

Thus, the statement $A_n = \frac{n}{n+1}$ is true for $n=1$ and $n=2$

Inductive step: (simple)

Assume that $A_n = \frac{n}{n+1}$ is true for each $n \in \{1, 2, 3, \ldots\}$, so

$$\frac{1}{1(2)} + \frac{1}{2(3)} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

let us prove that the statement is true for $n+1$ that

$$\frac{1}{1(2)} + \frac{1}{2(3)} + \ldots + \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)} = \frac{n}{n+1} + \frac{1}{(n+1)(n+2)}$$

$$= \frac{n(n+2)+1}{(n+1)(n+2)} = \frac{n^2+2n+1}{(n+1)(n+2)}$$

$$= \frac{(n+1)\cancel{(n+1)}}{\cancel{(n+1)}(n+2)}$$

$$= \frac{(n+1)}{(n+2)} = \frac{(n+1)}{(n+1)+1} = A_{(n+1)}$$

Thus the statement $A_n = \frac{n}{n+1}$ is true for $n+1$ which we have proved using simple induction.

4 Question 4 **19 / 20**

✓ **- 1 pts Issues with wording or presentation**

3 You only need to do 1

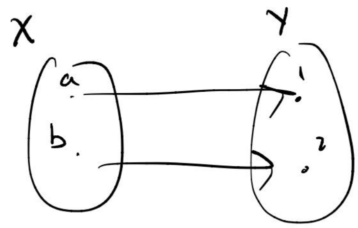
4 This what you are trying to prove, you can't just assume it. You mean something like "Assume $$A_n=n/(n+1)$$ for some $$n$$. We want to prove $$A_{n+1}=(n+1)/(n+2)$$"

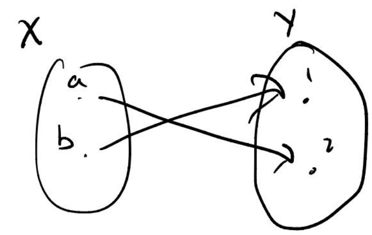
5. (20 points) Let $X = \{a, b\}$ and $Y = \{1, 2\}$. Consider the set

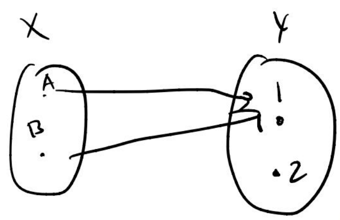
domain codomain

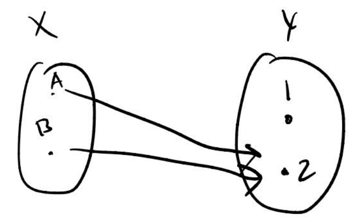
$$\mathcal{F} = \{f | f : X \to Y \text{ is a function from } X \text{ to } Y\}.$$

(a) (10 points) List all the elements of $\mathcal{F}$.









$\{(a,1),(b,2)\}$, $\{(a,2),(b,1)\}$, $\{(a,1),(b,1)\}$, $\{(a,2),(b,2)\}$

5.1 Part (a) 10 / 10

✓ **- 0 pts** **Correct**

(b) (5 points) How many elements of $\mathcal{F}$ are one-to-one?

F is a one-to-one function when

$\forall x_1, x_2 \in X$, $f(x_1) = f(x_2)$ then $x_1 = x_2$

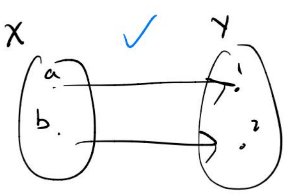
$F = \{(a,1),(b,2)\}, \{(a,2),(b,1)\}, \{(a,1),(b,1)\}, \{(a,2),(b,2)\}$

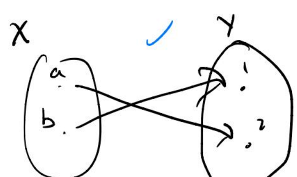
2 elements of F are one-to-one

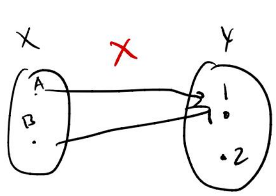
(c) (5 points) How many elements of $\mathcal{F}$ are onto?

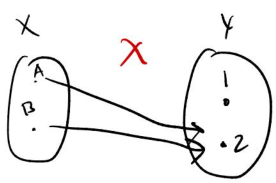
F is an onto function when

$\forall y \in Y$, $\exists x \in X$ s.t. $f(x) = y$









$F = \{(a,1),(b,2)\}, \{(a,2),(b,1)\}, \{(a,1),(b,1)\}, \{(a,2),(b,2)\}$

2 elements of F are onto

5.2 Part (b) 5 / 5

✓ **- 0 pts** **Correct**

(b) (5 points) How many elements of $\mathcal{F}$ are one-to-one?

F is a one-to-one function when

$\forall x_1, x_2 \in X$, $f(x_1) = f(x_2)$ then $x_1 = x_2$

$F = \{(a,1),(b,2)\}, \{(a,2),(b,1)\}, \{(a,1),(b,1)\}, \{(a,2),(b,2)\}$

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(c) (5 points) How many elements of $\mathcal{F}$ are onto?

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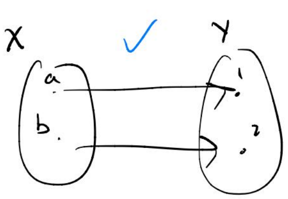

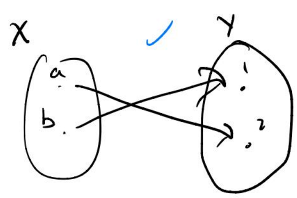

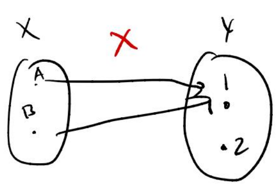

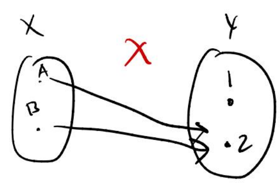

$F = \{(a,1),(b,2)\}, \{(a,2),(b,1)\}, \{(a,1),(b,1)\}, \{(a,2),(b,2)\}$

2 elements of F are onto

5.3 Part (c) **5 / 5**

✓ **- 0 pts** Correct

Math 61
Summer 2021
Midterm 1
07/01/2021
Time Limit: 24 Hours

Name: Mengan Wang

UID: 805605806

This exam contains 9 pages (including this cover page) and 5 questions. Total of points is 100. Make sure to write your answers in full detail, so that you may get the maximum possible points.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

- Please type the following statement in your handwriting, then sign and date below:

  "I hereby acknowledge that I am aware I may use my textbook, lecture notes and recordings during the exam and swear on my honor* as a Bruin *all* the answers I present belong solely to me, in thought and in writing."

I hereby acknowledge that I am aware I may use my textbook, lecture notes and recordings during the exam and swear on my honor* as a Bruin all the answers I present belong solely to me, in thought and in writing.

* See: `https://www.youtube.com/watch?v=UIXNOQ8ej9c&ab_channel=RGunn`

6 Academic Honesty Disclaimer **0 / 0**

✓ **- 0 pts** **Correct**