Instructions:

- You have from Friday 20 November 2020 at 00.00am to 11.59pm Pacific Time to solve this exam.
- Scan your solutions and upload them to Gradescope by Friday 20 November at 11.59pm Pacific Time. You should submit readable scans, and not pictures of your solutions. Please make sure to match the problems on the exam template with the respective parts in your solutions.
- This exam is open book, and you are allowed to use the textbook, and all resources from the lecture, or similar resources.
- You are not allowed to ask for help from thirds, nor give help to others taking this exam. Students suspected of academic dishonesty may be reported to the Dean of Students. This leads to a process which could end in suspension or dismissal.

Code of honor

Academic integrity is of the uttermost importance. By taking part in this evaluation, you are accepting the following code of honor:

I certify on my honor that I have neither given nor received any help, or used any non-permitted resources, while completing this evaluation.

Problem 1. (5 points) Can a graph have exactly five vertices of degree 1? Either draw an example of a graph satisfying this property or explain why such a graph can't exist.

Problem 2. (5 points) Let $x, y, w, z \in \mathbb{N}$ be such that $x \geq 0$, $y \geq 1$, $w > 0$ and $z \geq 3$. Using the stars and bars method, find the number of solutions of the following equation:

$$x + y + w + z = 20\,.$$

Problem 3. (5 points)
Let $X = \{0, 1, 2\}$, and let $G = (V, E)$ be the bipartite graph with vertex set $V = V_1 \cup V_2$, where $V_1 = X$ and $V_2 = \mathcal{P}(X)$. For any $v_1 \in V_1$ and $v_2 \in V_2$ there is an edge $\{v_1, v_2\} \in E$ if and only if v_1 is an element of v_2. Answer the following, by providing brief explanations:

1. Is G connected?
2. Suppose $v_1 \in V_1$. What is the degree of v_1?
3. Suppose $v_2 \in V_2$. What is the degree of v_2?

Problem 4. (5 points) Consider the recurrence relation

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + c_3 a_{n-3} + c_4 a_{n-4}$$

for $n \geq 4$. Show that if r is a root of the polynomial

$$x^4 - c_1 x^3 - c_2 x^2 - c_3 x - c_4,$$

then $a_n = r^n$ is a solution to the given recurrence relation.