Instructions:

- You have from Friday 23 October 2020 at 00.00am to 11.59pm Pacific Time to solve this exam.
- Scan your solutions and upload them to Gradescope by Friday 23 October at 11.59pm Pacific Time. You should submit readable scans, and not pictures of your solutions. Please make sure to match the problems on the exam template with the respective parts in your solutions.
- This exam is open book, and you are allowed to use the textbook, and all resources from the lecture, or similar resources.
- You are not allowed to ask for help from thirds, nor give help to others taking this exam. Students suspected of academic dishonesty may be reported to the Dean of Students. This leads to a process which could end in suspension or dismissal.

Code of honor

Academic integrity is of the uttermost importance. By taking part in this evaluation, you are accepting the following code of honor:

I certify on my honor that I have neither given nor received any help, or used any non-permitted resources, while completing this evaluation.

Problem 1. (4 points) Consider an arbitrary natural number $n \geq 2$. Let $A_1, \dots, A_n$ and C be arbitrary sets. Using mathematical induction, show that

$$\left(\bigcup_{i=1}^{n} A_i \right) \times C = \bigcup_{i=1}^{n} (A_i \times C) \; .$$

Problem 2. (4 points) Let R be a relation on a set X.

(a) Explain in words why the statement "R is anti-symmetric" is not the negation of the statement "R is symmetric". Provide examples to illustrate your explanation.

(b) Explain in words why the statement "R is anti-reflexive" is not the negation of the statement "R is reflexive". Provide examples to illustrate your explanation.

Problem 3. (3 points) Let n be a positive natural number. Let $X = \{i \in \mathbb{N} \colon 1 \leq i \leq n\}$. Denote by $\mathcal{P}(X)$ the power set of X, and let $\mathcal{P}^{\star}(X) := \mathcal{P}(X) \setminus \{\emptyset\}$ denote the set whose elements are subsets of X that are not empty. Consider the function

$$f \colon \mathcal{P}^{\star}(X) \to X$$

which sends each non-empty subset of X to its least element. For instance, $f(\{1,3\}) = 1$. For which values of n is f injective, surjective, or bijective? Carefully motivate your arguments.

Problem 4. (2 points) A teacher wants to arrange their 11 students in a single line. There are two students called Averie and Charlie in this class. How many ways are there for the students to line up so that Averie is first in line or Charlie is last?