Name: ____

UCLA ID Number: ______________________________________

Section letter: ____________

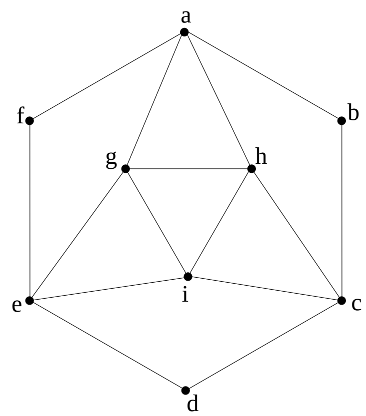
Math 61 : Discrete Structures
Midterm 2
Instructor: Ciprian Manolescu

You have 50 minutes.

No books, notes or calculators are allowed.
Do not use your own scratch paper.

1. (10 points) **Multiple choice:** Circle the right answers. You do NOT need to justify your answers.

Consider the following graph G:



(2 points) Is the graph G bipartite? Yes / $\boxed{No}$

(5 points) The graph G :

(A) has an Euler cycle and a Hamiltonian cycle;

$\boxed{(B)}$ has an Euler cycle but no Hamiltonian cycle;

(C) has a Hamiltonian cycle but no Euler cycle;

(D) has neither an Euler cycle nor a Hamiltonian cycle.

If the graph has an Euler cycle, write it down here (as an ordered sequence of vertices):

$$(a, f, e, g, a, h, g, i, e, d, c, i, h, c, b, a)$$

If the graph has a Hamiltonian cycle, write it down here (as an ordered sequence of vertices):

(3 points) Let A be the adjacency matrix of the graph G. For $n \geq 1$, let p_n denote the entry of A^n in the row labeled a and the column labeled c. Circle the correct answer:

(A) $p_1 = 0,\ p_2 = 1,\ p_3 = 1;$

(B) $p_1 = 0,\ p_2 = 2,\ p_3 = 2;$

(C) $p_1 = 0,\ p_2 = 2,\ p_3 = 2;$

$\boxed{(D)}$ $p_1 = 0,\ p_2 = 2,\ p_3 = 3$; paths of length 2 are abc and ahc; of length 3, we have $ahic, aghc$ and $agic$.

(E) $p_1 = 0,\ p_2 = 2,\ p_3 = 4.$

2. (12 points) Write down the answer to each question. *You do NOT need to justify your answers.*

(4 points) What is the number of integer solutions to

$$x_1 + x_2 + x_3 + x_4 = 100$$

with $x_1 \geq 1, x_2 \geq 2, x_3 \geq 3, x_4 \geq 4$?

Writing $y_i = x_i - i$, this is the same as asking for $y_1 + y_2 + y_3 + y_4 = 90$ with $y_i \geq 0$. The number of such solutions is

$$C(90 + 4 - 1, 4 - 1) = C(93, 3).$$

(4 points) Calculate:

$$\sum_{k=0}^{100} 5^k \cdot C(100, k).$$

Using the binomial formula, we get

$$(5 + 1)^{100} = 6^{100}.$$

(4 points) Let S_n be the number of strings of length n composed of the digits $0, 1$ and 2 that do not contain the pattern 00 (that is, do not have two consecutive 0's). Write down a recurrence relation for S_n.

Possible patterns to begin with: 1, 2, 01, 02. Hence:

$$S_n = 2S_{n-1} + 2S_{n-2}.$$

3. (8 points) Solve the recurrence relation

$$a_n = -2a_{n-1} - a_{n-2}$$

with the initial conditions $a_0 = 1, a_1 = 2$. *Show all your work.*

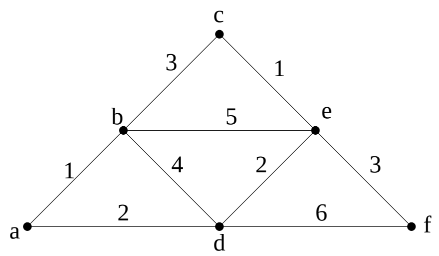
The associated quadratic equation is $t^2 + 2t + 1 = 0$ with double root $t = -1$. Therefore, the solution is of the form

$$a_n = (-1)^n(bn + d).$$

From $a_0 = 1$ we get $d = 1$ and then from $a_1 = 2$ we get $-b - d = 2$ so $d = -3$. Thus:

$$a_n = (-1)^n(1 - 3n).$$

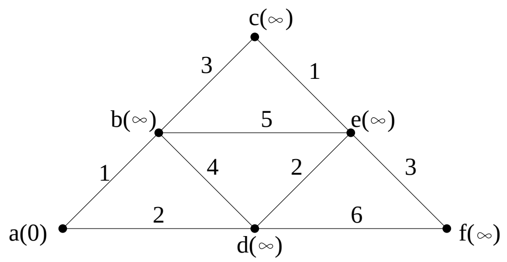
4. (10 points) Consider the following weighted graph:



Suppose you want to use Dijkstra's algorithm to find the length of the shortest path from a to f. Write down the initialization and the first two iterations of the algorithm. (You do NOT have to complete the whole algorithm.) At each step, write down if f is in T, what is the current node, what is the new set of unvisited vertices T, then which nodes change labels and how.

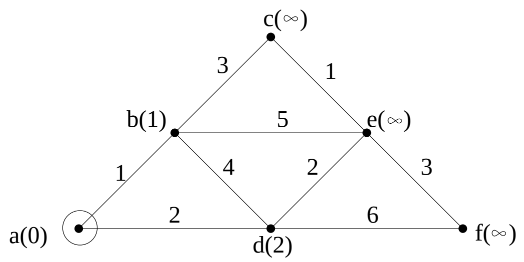
Also, please circle the visited labels at each step (the ones not in T), and write down the label at each vertex in paranthesis.

Initialization:



T={a,b,c,d,e,f}

First iteration:



Is f in T? Yes

current node: a

T={b,c,d,e,f}

changes in labels: d: min(∞,2)=2

b: min(∞,1)=1

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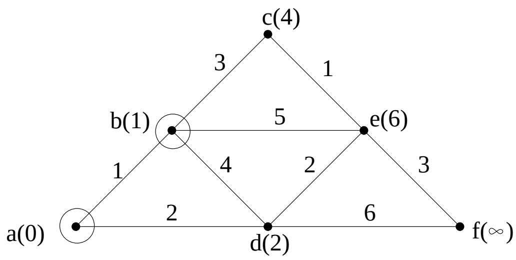
Second iteration:



Is f in T? Yes

current node: b

T={c,d,e,f}

changes in labels:

c: min(∞,1+3)=4
d: min(2,1+4)=2
e: min(∞,1+5)=6