Math 61, Lec 1
Winter 2016
Exam 1
1/25/16
Time Limit: 50 Minutes

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Discussion Section: 1E

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing.

You may *not* use books, notes, or any calculator on this exam.

If your answer contains a number that is impossible to simplify without the use of a calculator, such as e^3, $\ln(3)$ or $\sin(3)$, you may leave answers in terms of e, ln, or trig functions.

Partial credit will only be awarded to answers for which an explanation and/or work is shown.

Please attempt to organize your work in a reasonably neat and coherent way, in the space provided. If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	7	7
2	18	17
3	25	25
4	25	25
5	25	17
Total:	100	91

1. (7 points) Negate the following implication:
"If you are on the wait list, then you will be enrolled in the class."

p = on wait list

q = enrolled in the classs

$\neg(p \to q) = p \wedge \neg q$

you are on the waitlist and you won't be enrolled in the class.

2. (18 points) (a) Let X be a set with n elements. How many different relations on X are there?

2^{n^2}

(b) Let X be a set with n elements. How many different relations on X are there that are not reflexive?

$A =$ ~~###~~ total relations on X $= n^n$

$B =$ total relations that are reflexive $=$

$B = 2^{n^2-n}$

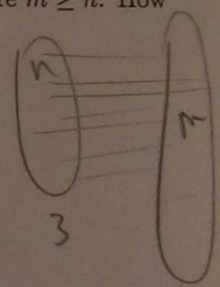
relations on X that are not reflexive $= A - B = 2^{n^2} - 2^{n^2-n}$

$= 2^{n^2}[1-2^{-n}]$

(c) Let X be a set with n elements and let Y be a set with m elements, where $m \geq n$. How many different one-to-one functions from X to Y are there?

$m \cdot (m-1) \cdots (m-n+1))$

$= \frac{m!}{(m-n\cancel{+1})!}$



2 3

$m-n$

$3 \cdot 2 \cdot (1)$

3. (25 points) Use mathematical induction to prove that $1+3+5+\cdots+(2n+1)=(n+1)^2$ for every integer $n \geq 0$.

Basis Step:

$n=0$

$1 = (0+1)^2$

$1 = 1$ so it is true for the base case $n=0$

Inductive Step: we assume it is true for n. Now we try to prove it for $n+1$

$1+3+5+\cdots+2n+1 + 2(n+1)+1 = (n+2)^2$

$2n+3$

so $F(n) + a_{n+1} = F(n+1)$

$(n+1)^2 + 2n+3 = (n+2)^2$

$n^2+2n+1+2n+3 = n^2+4n+4$

$n^2+4n+4 = n^2+4n+4$

as both sides the equation are equal, we have prove n for $n+1$.

Now according to the principle of mathematical induction, we have also proven it for n.

for all $n \geq 0$.

$\blacksquare$

4. (25 points) Let X be a set. We define the *power set* $\mathcal{P}(X)$ to be the set of all subsets of X. For example, if $X = \{1, 2\}$, then $\mathcal{P}(X) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$.

Define a relation R on $\mathcal{P}(\mathbb{Z})$ by $(S, T) \in R$ if and only if $S \subseteq T$, for any sets S and T in $\mathcal{P}(\mathbb{Z})$. Prove that R is a partial order.

Reflexive: so $(S,S) \in R$ if & only if $S \subseteq S$ which is true as a set is a subset of itself. So R is reflexive.

Anti-symmetric: so if $(S,T) \in R$ if and only if $S \subseteq T$ then $(T,S) \in R$ if and only if $T \subseteq S$ but in order for $S \subseteq T$ and $T \subseteq S$ to hold true $T = S$. which is the principle behind anti-symmetric property. Hence R is anti-symmetric.

Transitive: so if $(S,T) \in R$ and $(T,U) \in R$ only if $S \subseteq T$ and $T \subseteq U$.
If the above is true. Then $S \subseteq U$ as S is a subset of T which is a subset of U.
This means $(S,U) \in R$ holds true. This makes R a transitive.

Hence R is partial order.

5. (25 points) As in the previous problem, for any set X, we define the *power set* $\mathcal{P}(X)$ to be the set of all subsets of X.
Consider the sets $X = \{1, 2, 3\}$ and $Y = \{4, 5, 6\}$.
Define a function $f : \mathcal{P}(X) \times \mathcal{P}(Y) \to \mathcal{P}(X \cup Y)$ by $f((S,T)) = S \cup T$, for $S \in \mathcal{P}(X)$ and $T \in \mathcal{P}(Y)$. Prove that f is a bijection.

one-to-one: so if $f(x) = f(y)$

then $x = y$

as it can be seen $s \in X$ and $s \notin Y$

and $t \in Y$ and $t \notin X$

so for set S_1, T_1 and S_2 and T_2

if $f((S_1, T_1)) = f((S_2, T_2)) = S_1 \cup T_1 = S_2 \cup T_2$

$(S_1, T_1) = (S_2, T_2)$ must be true

$S_1 = S_2$ as $X \cap Y = \phi$

$T_1 = T_2$

onto: so for $\forall y, \exists x$ such that $f(x) = y$

In this case, $\forall A \in S \cup T$, $\exists B_1 \in S\ \exists B_2 \in T$ such that

$f((B_1, B_2)) = A$

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so for $x \in P(X)$ and $y \in P(Y)$

$x \cup y \in P(X \cup Y)$ is true as they both have the same set elements that they can contain.

Hence it is onto.