Med: 78

Math 61, Lec 1 Winter 2016 Exam 1 1/25/16

Nar Na

Time Limit: 50 Minutes

Discussion

This exam contains 6 pages (including this cover page) are missing.

You may not use books, notes, or any calculator on this exam.

If your answer contains a number that is impossible to simplify without the use of a calculator, such as e^3 , $\ln(3)$ or $\sin(3)$, you may leave answers in terms of e, \ln , or trig functions.

Partial credit will only be awarded to answers for which an explanation and/or work is shown.

Please attempt to organize your work in a reasonably neat and coherent way, in the space provided. If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score	
1	7	7	
2	18	17	
3	25	22	
4	25		25
5	25	25	
Total:	100	96	

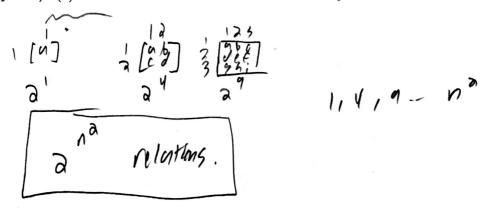
1. (7 points) Negate the following implication: "If you are on the wait list, then you will be enrolled in the class."

I am on the want 11st, and not enalled in the class.

P = on walt/1st

q = enelled in class

2. (18 points) (a) Let X be a set with n elements. How many different relations on X are there?



(b) Let X be a set with n elements. How many different relations on X are there that are not reflexive?

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0$$

(c) Let X be a set with n elements and let Y be a set with m elements, where $m \ge n$. How many different one-to-one functions from X to Y are there?

$$\begin{cases} 1 - n^3 & \text{firm} \\ 1$$

3. (25 points) Use mathematical induction to prove that $1+3+5+\cdots+(2n+1)=(n+1)^2$ for every integer $n \ge 0$.

Base case:

$$N = C$$
: $2(C) + 1 = 1$: $(C+1)^{\frac{3}{2}} = 1$
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4. (25 points) Let X be a set. We define the *power set* $\mathcal{P}(X)$ to be the set of all subsets of X. For example, if $X = \{1, 2\}$, then $\mathcal{P}(X) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$.

Define a relation R on $\mathcal{P}(\mathbb{Z})$ by $(S,T) \in R$ if and only if $S \subseteq T$, for any sets S and T in $\mathcal{P}(\mathbb{Z})$. Prove that R is a partial order.

Refletive: Let set X EP(ZD) be additing, X \(\times \), so \(\times \) \(\times \).

anti-symptic: Let Sets $X, Y \in POZ)$ be orbitrary.

Assum $(X,Y) \cap (Y,X) \in R$. Thun, $X \subseteq Y$, and $Y \subseteq X$. By definition

of set equally, X = Y.

Transitive. Let Sets $X,Y,Z\subseteq P(Z)$ be ablown,

Assum $(X,Y)^{\Lambda}(Y,Z)\in R$. Then, $X\subseteq Y$,

and $Y\subseteq Z$. Let $Y\in X$ be ablown,

Because $X\subseteq Y$, $X\in Y$. Then, because $Y\subseteq Z$, $X\in Z$, G_{C} , $Y\times GY$, $X\in Z$. Therefore, $X\subseteq Z$, G_{C} $Y\times GY$, $X\in R$.

Reflexiv, anti-symmetry transition => Partial drafer

5. (25 points) As in the previous problem, for any set X, we define the power set $\mathcal{P}(X)$ to be the set of all subsets of X.

Consider the sets $X = \{1, 2, 3\}$ and $Y = \{4, 5, 6\}$. Define a function $f: \mathcal{P}(X) \times \mathcal{P}(Y) \to \mathcal{P}(X \cup Y)$ by $f((S,T)) = S \cup T$, for $S \in \mathcal{P}(X)$ and $T \in \mathcal{P}(Y)$. Prove that f is a bijection.

Let A, B & P(X) and CIDE P(4) be option, +(A,C) = + (B,D), AUC = BUD. Let x EA to obstray. Beauty X + A, x & A VE, a and x & BUD. But, bucause XAY= \$, X\$ D, So XGB. So, A & B. By a Charsing Carbitrary X & B instead, by similar argumt BCA. Sc, A=B. Ushg Egine layic for and stay yEC, y E/AUC = 7 y & BUD, and because y & B, y & D, so C & Drund 01/50 D&C. Sol Geranice A=B1C=D, Let CEP(XVY) be orbitran Let a EC

be artition, By definition, at XV a & Y. So,

for some A & PCD, B& PCK), 9 6 A V a & B.

Sc, AUB CC. Then, for som b & C, 1 for some DEPC+), E EPCX DED 168 E. S., CE DUE. Because 96/ and 6 & T on not mutually excluster 月A, DE PCK) 1 B, 日 EPCY) AUR = DVG. Sr, for CUT & CE PCXUY) - A SEPCY), BEPCY) F (AB) = C