Q1 Honor Code + Rules

1 Point

Read the rules section. They have been modified.

Please read the following Honor Code and upload your signature verifying that you have read it and agree to it. You will receive a 0 on the exam otherwise.

"I certify on my honor that I have neither given nor received any help, or used any non-permitted resources, while completing this evaluation. I have spoken to no one about this exam and have not posted the exam (or any of the question below) to any online forums. I have read the exam rules on CCLE. I will draw a small picture after my signature. Moreover, the solutions I have posted are entirely my work and in my own handwriting."

Rules:

1. You may use your books, notes, CCLE, and the internet. **However, you may not use websites such as Chegg**. If any of your solutions are copied from Chegg, you will be given a 0 on the exam and be reported to the Dean of Students. You may not discuss the exam with anyone outside of myself (Mr. Gannon) and the TAs.
2. You may write on a tablet but do not use LaTeX (and especially not Word). Your answers must be in your own handwriting.

Q2 Pigeonhole

10 Points

1. Suppose Class A has 200 people. Assume we have a large box containing green, yellow, and orange balls and every student in Class A chooses 6 balls. Show that there are (at least) 8 students who draw the same number of each color of balls. To be clear, we are saying that there exists a set of 8 students such that any two students in this set have the the same number of green balls, the same number of yellow balls, and the same number of orange balls.
2. Assume there is another class, Class B. Every student in class B writes down a word of length 6 using the letters $\{a, b, c\}$. For instance, Student G writes '$babacc$' while Student K writes '$aaaaab$'. Determine the minimum number of students Class B needs to have so that at least two students write the same word.

Q3 Recurrence

10 Points

Let $\{s_n\}_{n=1}^{\infty}$ be a sequence of real numbers satisfying the following:

1. $2s_n - 14s_{n-1} + 24s_{n-2} = 0$ for all $n \geq 3$.
2. $s_1 = s_2 = 1$.

What is the sequence $\{s_n\}_{n=1}^{\infty}$? Your answer should be an explicit formula for s_n in terms of n.

Show all of your work.

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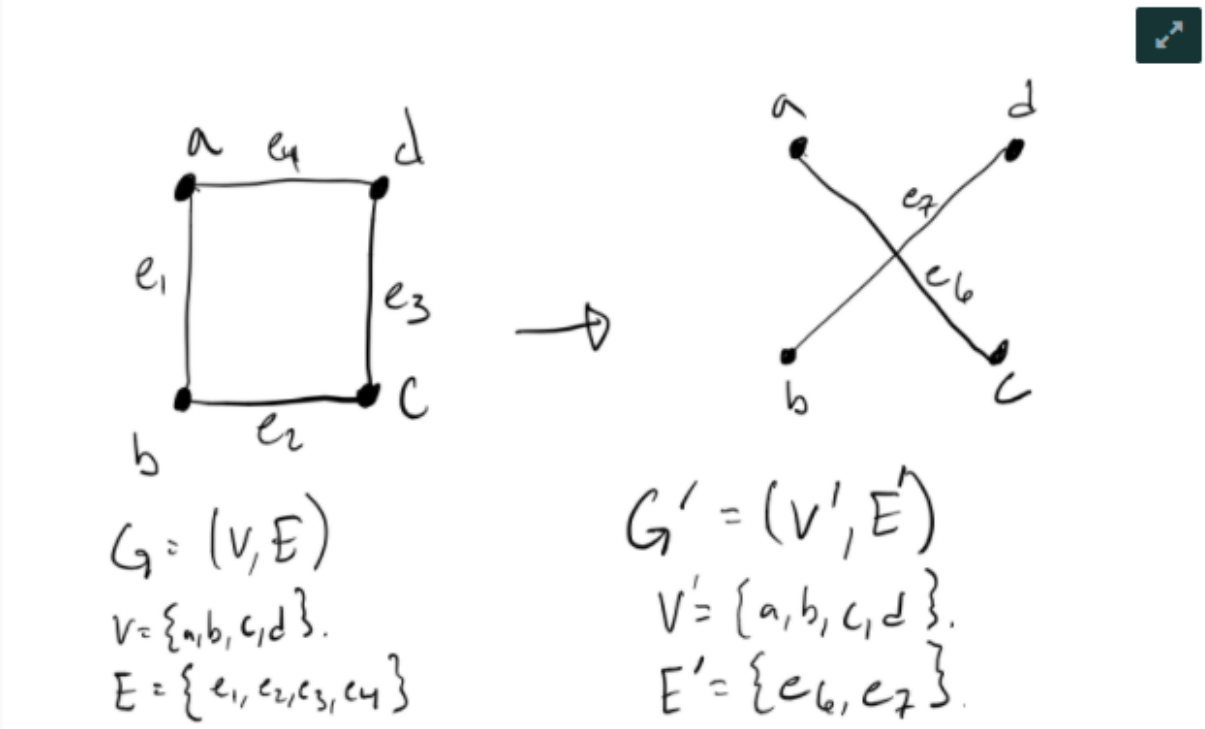
Q4 Graphs 1

10 Points

Let $G = (V, E)$ be a **simple graph**. We construct the opposite graph $G' = (V', E')$ as follows:

1. The vertex set stays the same $V = V'$.
2. For two distinct vertices $v, w \in V'$, there is precisely one edge $e \in E'$ connecting these vertices if and only if there is **not** an edge in E connecting v and w.

For example, if $G = (V, E)$ is the graph on the left in the picture below, then G' is the graph on the right.



1. Does there exists a simple graph G on 6 vertices such that both the graph G and its opposite graph G' are connected? If yes, provide an example, if no provide a proof.
2. Does there exists a simple graph G on 6 vertices such that both G and its opposite graph G' contain an Euler cycle? If yes, provide an example, if no provide a proof.

Hint for part 2: Degrees of vertices.

Q5 Graphs 2

10 Points

Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. We define the following **simple graph** $G = (V, E)$ where V is the collection of subsets of X of size exactly 2 (i.e. $V = \{A : A \subseteq X \text{ and } |A| = 2\}$).

For example, $\{1, 2\}$, $\{2, 5\}$, and $\{4, 8\}$ are an elements in V, but $\{1, 2, 3\}$ is not.

For any two distinct vertices $v, w \in V$, there is an edge between v and w if and only if $v \cap w \neq \emptyset$.

For example, there is an edge between the vertices $\{1, 2\}$ and $\{2, 3\}$, but there is no edge between the vertices $\{1, 2\}$ and $\{3, 4\}$.

1. Determine if G is connected. Prove your claim.
2. What is the degree of the vertex $\{1, 2\}$? Prove your claim.
3. Does this graph have an Euler cycle? Prove your claim.