Please read the following Honor Code and upload your signature verifying that you have read it and agree to it. You will receive a $0$ on the exam otherwise. "I certify on my honor that I have neither given nor received any help, or used any non-permitted resources, while completing this evaluation. I have spoken to no one about this exam and have not posted the exam (or any of the question below) to any online forums. I have read the exam rules on CCLE. I will draw a small picture after my signature. Moreover, the solutions I have posted are entirely my work and in my own handwriting."

**You may write on a tablet but do not use LaTeX (and especially not Word).**
**Your answers must be in your own handwriting.**
**Please upload a single pdf per question.**

## **Q2** Induction

10 Points

Consider the set $X = \{a,b,c\}$. Let $S(n)$ be the set of strings of length $n$ constructed from the elements in $X$.
For example, $abbc$ and $aaaa$ are a strings of length 4 and so $abbc \in S(4)$ and $aaaa \in S(4)$.
Prove by induction that for each $n \in \mathbb{N}$, $|S(n)| = 3^{n}$.
Make sure to explicitly state your base case and induction hypothesis.

## **Q3** Relations

10 Points

Let $X = \{x_1,x_2,x_3, x_4, x_5\}$ and consider the relation $R = \{(x_1,x_2),(x_4,x_3)\}$

### Q3.1

5 Points

Give an example of a relation $R_{1}$ such that $R \subseteq R_{1}$ and $R_{1}$ is an equivalence relation on $X$. Explain why your example works.
Note: You may use diagraphs to help your explanation, but please also give an answer in set notation, i.e. $R_{1} = \{\ldots$

Q3.2
5 Points

Give an example of a relation R_{2}$R_2$ such that R \subseteq R_{2}$R \subseteq R_2$ and R_{2}$R_2$ is a partial order on X$X$. Explain why your example works.
Note: You may use diagraphs to help your explanation, but please also give an answer in set notation, i.e. R_{2} = \{...$R_2 = \{ \ldots$

## **Q4** Sequences
10 Points

Let S=\{ s \mid s:\{1,2,3\} \to \{1,2,3\}\}$S=\{s \mid s:\{1,2,3\} \to \{1,2,3\}\}$. Notice that each element of S$S$ is a sequence and so it makes sense to ask whether an element in S$S$ is increasing, decreasing, etc.
(*Part of this question is interpreting what is written.*)

1. Find |S|$|S|$.
2. How many elements in S$S$ are increasing?
3. How many elements in S$S$ are decreasing?
4. How many elements in S$S$ are non-increasing or (inclusively) non-decreasing?

Show all your work and explain your answers.

## **Q5** Permutations and Combinations
10 Points

The pet store down the street from my apartment has decided to host a parade. The pet store has 5 dogs, 3 cats, and 1 rooster. As we all know, cats, dogs and rooters are mortal enemies and so the parade has the following conditions:

1. All animals are in a single line.
2. The set of cats cannot be split up and the set of dogs cannot be split up. Another way to say this is that all the cats must remain together and all dogs need to remain together. (For example, C_1,C_2,C_3,R_1,D_1,D_2,D_3,D_4,D_5$C_1,C_2,C_3,R_1,D_1,D_2,D_3,D_4,D_5$ and C_3,C_2,C_1,D_2,D_3,D_4,D_5,D_1,R_1$C_3,C_2,C_1,D_2,D_3,D_4,D_5,D_1$

,$R_1$ are acceptable parades, but C_1,C_2,C_3,D_2,D_3,R_1,D_4,D_5,D_1$C_1,C_2,C_3,D_2,D_3,R_1,D_4,D_5,D_1$ is **not** since the rooster splits up the set of dogs into two chunks. Moreover, the parade beginning C_1,C_2,D_1,C_3…$C_1,C_2,D_1,C_3\ldots$ is **not** acceptable since (for instance) the cats have been separated from one another).

3. All these animals have distinct personalities, so different orderings count as different parades (For example, a parade starting D_1,D_2…$D_1,D_2\ldots$ is different from a parade starting D_2,D_1…$D_2,D_1\ldots$.)

How many possible parades are there which satisfy the conditions above? You must explain your work. Use *principles* and/or theorems from class and state them explicitly when you use them.