19F-MATH61-1 Midterm II Blue

HENRY MACARTHUR

TOTAL POINTS

38 / 40

QUESTION 1

1 Recurrence 8 / 8

- ✓ + **8 pts** **All parts correct**
- + **3 pts** Set up and solved characteristic polynomial correctly
- + **3 pts** Recognized correct form of solution
- + **2 pts** Solved for initial conditions correctly
- - **1 pts** Small error
- + **1 pts** Incorrect form but correct continuing work
- + **0 pts** No credit

QUESTION 2

Weighted graph 12 pts

2.1 Isomorphism 3 / 3

- ✓ + **3 pts** **Correct**
- - **1 pts** No explanation
- + **0 pts** Incorrect
- - **1 pts** Phrasing unclear

2.2 Vertex circled? 2 / 2

- ✓ + **2 pts** **Correct**
- + **0 pts** Incorrect

2.3 Shortest path 3 / 3

- ✓ + **3 pts** **Correct**
- + **0 pts** Incorrect
- - **1 pts** Incorrect, incomplete, or missing work
- + **1 pts** Work shown, but with a small mistake

2.4 Euler cycle 3 / 4

- ✓ + **2 pts** **Says that there is an Euler cycle**
- ✓ + **1 pts** **Says that every vertex has even degree**
- + **1 pts** Says that graph is connected
- + **0 pts** Incorrect
- + **2 pts** Gives an Eulerian cycle

QUESTION 3

3 Pigeonhole Principle 7 / 8

- + **1 pts** There are serious issues with the proof.
- + **2 pts** There are some issues with the proof.
- + **3 pts** Fairly complete and correct proof.
- ✓ + **4 pts** **Nearly complete and correct proof.**
- + **5 pts** Well-written and well-reasoned proof, with complete sentences and correct logic.
- + **1 pts** Setup of the pigeonhole argument is unclear or flawed.
- + **2 pts** The set up of the pigeonhole argument is mostly complete and correct.
- ✓ + **3 pts** **Set up pigeonhole argument correctly: described pigeons, pigeonholes, and how to assign pigeons to holes.**
- + **5 pts** Essentially correct argument, but did not use the pigeonhole principle (the problem explicitly asks for this).

1 802, last person in line could be USC

2 True, but you need to spell it out a bit more. We know 3 of the 1800 go to the same spot, but why must there be one from each of the +0, +1, and +2 sequences

3 consecutive?

4 This crossed out argument would have worked if you reversed the role of UCLA and USC

QUESTION 4

Incidence matrix 12 pts

4.1 Bipartite 3 / 3

- ✓ + **1 pts** **Correct picture**
- ✓ + **2 pts** **Correctly determined whether the graph is bipartite**

+ **0 pts** Incorrect

4.2 Paths 5 / 5

+ **3 pts** Correct answer
+ **2 pts** Clear calculation and/or reasoning

Matrix Approach

✓ + **1 pts** **Correct adjacency matrix**

✓ + **1 pts** **Idea to find 3,3 entry of 6th power of matrix**

✓ + **3 pts** **Correct calculation**

+ **2 pts** Mostly correct calculation
+ **1 pts** Somewhat correct calculation

+ **0 pts** Incorrect

4.3 Hamiltonian 4 / 4

Answer

✓ + **2 pts** **Correct answer**

+ **0 pts** Incorrect answer

Reasoning

✓ + **2 pts** **Clear and correct reasoning**

+ **1 pts** Needs more/better explanation
+ **0 pts** Incorrect reasoning

Could be organized better on the page

5 ?

DO NOT OPEN THIS EXAM UNTIL YOU ARE INSTRUCTED TO DO SO!

Class:	Math 61, Lecture 1
Instructor:	Jonathan Rubin
Exam:	Midterm II
Date:	18 November 2019
Time:	11:00 AM – 11:50 AM

THIS IS A CLOSED BOOK EXAM. NO OUTSIDE AIDS, SUCH AS NOTES, TEXTBOOKS, CALCULATORS, OR CELLPHONES ARE PERMITTED.

First and Last Name: Henry MacArthur

Student ID Number: 709096169

Section and Teaching Assistant: 1A Soukup

I understand that this is a closed book exam. I certify that the following work is mine alone, and I pledge that I have neither given nor received unauthorized assistance on this test.

Signature: Henry MacArthur

Instructions: This is a 50-minute exam. It consists of four problems, and there is an extra piece of scratch paper at the end. Please write your answers in the space provided. If you run out of room, then please continue onto the back of the page and indicate clearly that you have done so. **Good luck!**

Question	Points	Score
1	8	
2	12	
3	8	
4	12	
Total:	40	

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1. (8 points) Let $s_0, s_1, s_2, \ldots$ be the sequence such that

   (i) $s_0 = 2$, $s_1 = 12$, and

   (ii) $s_n = 8s_{n-1} - 16s_{n-2}$ for all $n \geq 2$.

   Find a formula for s_n. Please circle your answer and show your work.

$s_n = 8s_{n-1} - 16s_{n-2}$

$t^2 = 8t - 16$

$t^2 - 8t + 16 = 0$

16

-4 -4

-8

$(t-4)^2 = 0$

$t = 4, 4$

have repeated root $r = 4$.

$s_n = Ar^n + Bnr^n$

$s_n = A(4)^n + Bn(4)^n$

$s_0 = 2$

$s_1 = 12$

$s_0 = A(4)^0 + B(0)(4)^0 = 2$

$A = 2$

$s_1 = A(4)^1 + B(1)(4)^1 = 12$

$4A + 4B = 12$

$4(2) + 4B = 12$

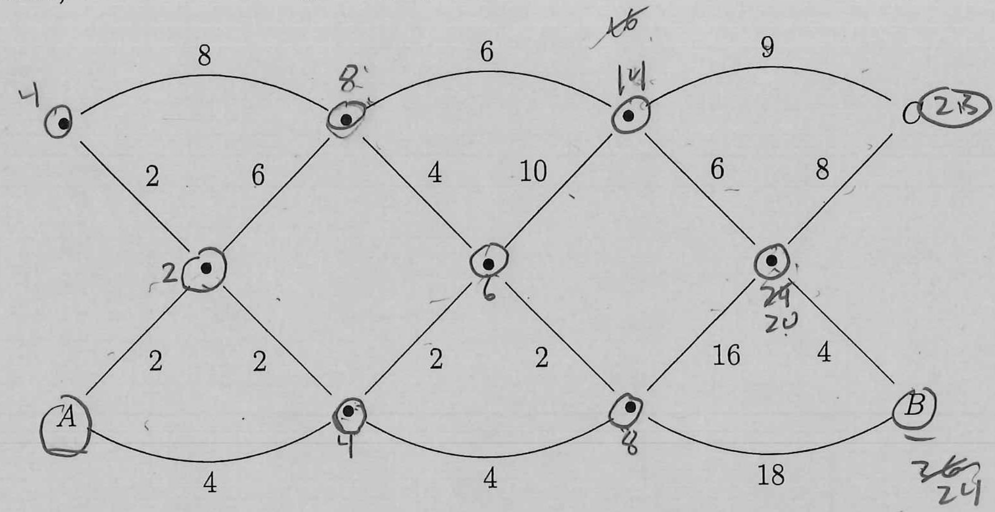
$8 + 4B = 12$

$4B = 4$

$B = 1$

$s_n = 2(4)^n + (1)(n)(4)^n$

2. (12 points) Let G be the weighted graph below.



(a) (3 points) Is G isomorphic to the complete graph on 10 vertices? Briefly explain.

no, in the complete graph on 10 vertices, each vertex has a degree of 9 as it has an edge between itself and all 9 other vertices. In G, all vertices have degree ≤ 4, since degree of vertices in G does not match degree of vertices in K_{10}, they are not isomorphic

(b) (2 points) Suppose we calculate the length of the shortest path from A to B using Dijkstra's algorithm. Is vertex C circled at the end? Circle one: (Yes) No

(c) (3 points) What is the length of the shortest path from A to B? Write your answer below, but show your work on the graph above.

24

(d) (4 points) Does the graph G have an Euler cycle? Justify your answer.

yes, each vertex in graph G has even degree as each vertex has degree 2 or 4.

3. (8 points) Suppose that 200 UCLA students and 600 USC students form a single line of 800 people to enter the Los Angeles Coliseum. Use the pigeonhole principle to prove that there are three consecutive USC students in the line.

total 600 usc students a_i

there is a total numbered of a_1 600 students a_{600}

4

~~if we make a line of 600 usc students,~~ to add ucla students
there ~~are 601 spots between them~~

so ~~having~~ 601 spots and 200 students, we have

$\lceil \frac{600}{200} \rceil$ = 3 we know 3 usc students must be consecutive.

3

all possible next to 1 student

→ 600 possible

- $a_1, a_2, a_3, \ldots a_{600}$
- $a_1+1, a_2+1, a_3+1, \ldots a_{600}+1$
- $a_1+2, a_2+2, a_3+2, \ldots a_{600}+2$

establishes 3 in a row

so total 1800 and USC students range from 1 to (600+2)

1

range from 1 to 602 spots for USC students and we have 1800 total spots. using pigeon hole by $\lceil \frac{1800}{602} \rceil = 3$ we know at least 3 USC students must be in a row in line.

2

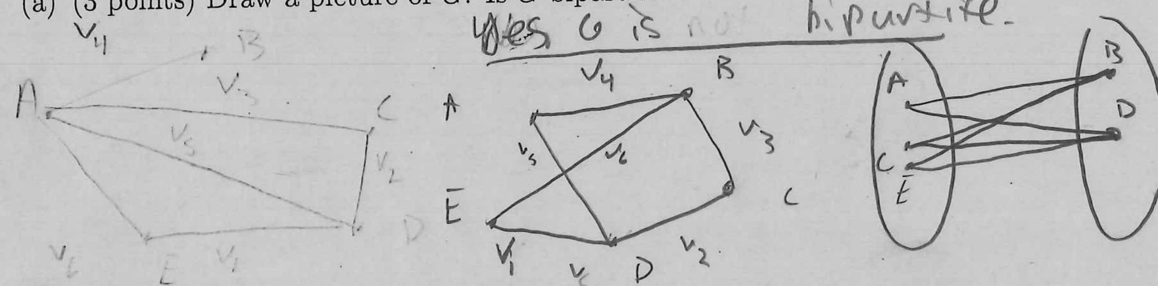
4. (12 points) Suppose that G is a graph whose incidence matrix is

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} v_1\ v_2\ v_3\ v_4\ v_5\ v_6 \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}.$$

This means that every row corresponds to a vertex in G, every column corresponds to an edge in G, and the (i, j)-entry is 1 if and only if edge j is attached to vertex i.

(a) (3 points) Draw a picture of G. Is G bipartite?

~~yes~~ G is ~~not~~ bipartite.



(b) (5 points) Let v be the vertex of G that corresponds to the third row of the matrix above. How many length six paths are there from v to itself? Show your work.

work on back as well

$$A = \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array}\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$[0\ 1\ 0\ 1\ 0] \cdot A$

$= [2\ 0\ 2\ 0\ 2] \cdot A$

$= [0\ 6\ 0\ 6\ 0] \cdot A$

$= [12\ 0\ 12\ 0\ 12] = A^4$

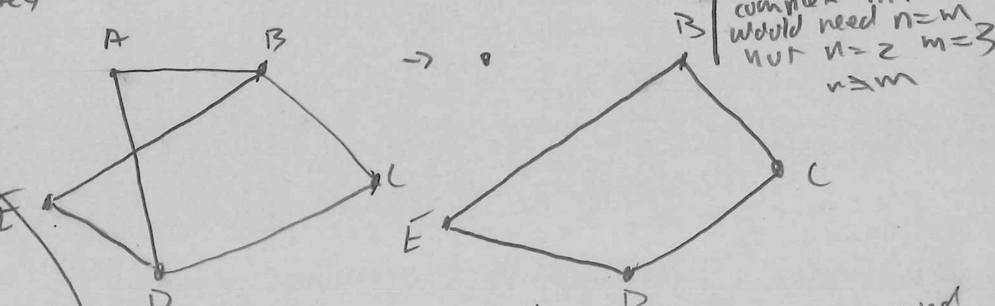
$A^5 = [0\ 36\ 0\ 36\ 0]$

$A^6 = [72\ 0\ 72\ 0\ 72]$

36 + 36 = 72

72

(c) (4 points) Does the graph G contain a Hamiltonian cycle? Justify your answer.



Also G is complete bipartite, would need n=m but n=2 m=3, n≠m

this graph is closed and since each vertex in it has degree 2, there is no way to connect the last two vertices as some vertices would have degree 3, meaning we cannot create a hamiltonian cycle.

and, seeing that vertex C and E have degree 2, their two edges must exist in the hamiltonian path, doing this, we get the graph above,

Extra scratch paper.

$$A \cdot A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} =$$

$$A^2 \cdot A^2 = \begin{bmatrix} 2 & 0 & 2 & 0 & 2 \\ 0 & 3 & 0 & 3 & 0 \\ 2 & 0 & 2 & 0 & 2 \\ 0 & 3 & 0 & 3 & 0 \\ 2 & 0 & 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & 2 & 0 & 2 \\ 0 & 3 & 0 & 3 & 0 \\ 2 & 0 & 2 & 0 & 2 \\ 0 & 3 & 0 & 3 & 0 \\ 2 & 0 & 2 & 0 & 2 \end{bmatrix}$$

$A^2 \to$ 3rd column

A^4 – 3rd row

$$[12 \;\; 0 \;\; 12 \;\; 0 \;\; 12] \cdot \begin{bmatrix} 12 \\ 0 \\ 12 \\ 0 \\ 12 \end{bmatrix} = [72 \;\; 0 \;\; 72 \;\; 0 \;\; 72]$$

24
24
24
72