DO NOT OPEN THIS EXAM UNTIL YOU ARE INSTRUCTED TO DO SO!

Class: Math 61, Lecture 1
Instructor: Jonathan Rubin
Exam: Midterm II
Date: 18 November 2019
Time: 11:00 AM – 11:50 AM

THIS IS A CLOSED BOOK EXAM. NO OUTSIDE AIDS, SUCH AS NOTES, TEXTBOOKS, CALCULATORS, OR CELLPHONES ARE PERMITTED.

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I understand that this is a closed book exam. I certify that the following work is mine alone, and I pledge that I have neither given nor received unauthorized assistance on this test.

Signature: Edward Deng

Instructions: This is a 50-minute exam. It consists of four problems, and there is an extra piece of scratch paper at the end. Please write your answers in the space provided. If you run out of room, then please continue onto the back of the page and indicate clearly that you have done so. **Good luck!**

Question	Points	Score
1	8	
2	12	
3	8	
4	12	
Total:	40	

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1. (8 points) Let $s_0, s_1, s_2, \ldots$ be the sequence such that

   (i) $s_0 = 2$, $s_1 = 12$, and

   (ii) $s_n = 8s_{n-1} - 16s_{n-2}$ for all $n \geq 2$.

   Find a formula for s_n. Please circle your answer and show your work.

$S_n = 8S_{n-1} - 16S_{n-2} \to r^n = 8r^{n-1} - 16r^{n-2} \to r^2 - 8r + 16 = 0$

$(r-4)^2 = 0$

$r = 4$

single root

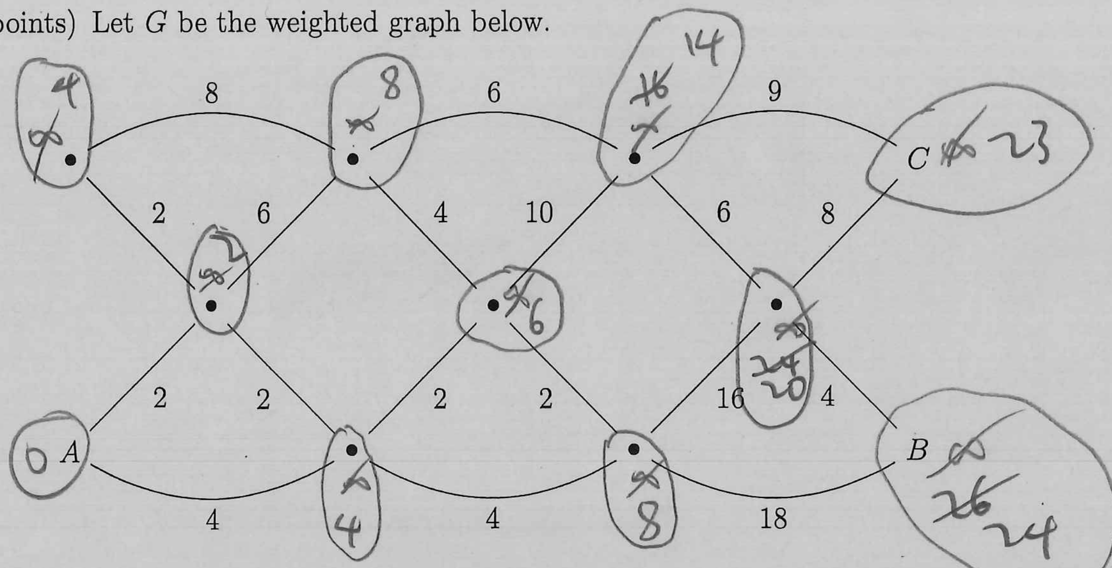
$Ar^n + nBr^n = S_n$ (for single root)

$S_0 = 2 = A$ $\quad S_1 = 12 = 4A + 4B$

$B = \frac{12 - 4A}{4} = 3 - A = 3 - 2 = 1$

$S_n = 2(4^n) + n(4^n)$

2. (12 points) Let G be the weighted graph below.



(a) (3 points) Is G isomorphic to the complete graph on 10 vertices? Briefly explain.

No, G isn't isomorphic to K_{10}. For one thing, G has 11 vertices, so its vertex set can't be in bijection with the vertex set of K_{10}. Also, the degrees of the vertices in G don't match up with the degrees of the vertices in K_{10}.

(b) (2 points) Suppose we calculate the length of the shortest path from A to B using Dijkstra's algorithm. Is vertex C circled at the end? Circle one: Yes **(No)**

(c) (3 points) What is the length of the shortest path from A to B? Write your answer below, but show your work on the graph above.

The length of the shortest path is 24.

(d) (4 points) Does the graph G have an Euler cycle? Justify your answer.

G has an Euler cycle. A graph has an Euler cycle if it is connected and every vertex has even degree. G is connected, and every vertex of G has even degree, so G does have an Euler cycle.

3. (8 points) Suppose that 200 UCLA students and 600 USC students form a single line of 800 people to enter the Los Angeles Coliseum. Use the pigeonhole principle to prove that there are three consecutive USC students in the line.

Let the 600 USC students be "pigeons".
Let the 200 UCLA students be "holes", in the sense of dividers between USC students (201 holes).
If you were to spread out the USC "pigeons" as much as possible among the 201 UCLA "holes", you would have two point something pigeons per hole. In reality, this translates to some holes with 2 people and at least one hole with 3 people. The point here is that there is a hole with 3 people (3 consecutive USC students). So, by the pigeonhole principle, there are 3 consecutive USC students in the line.

4. (12 points) Suppose that G is a graph whose incidence matrix is

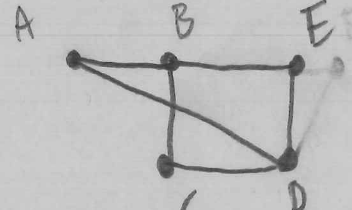
$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

adjacency:

	A	B	C	D	E
A	0	1	0	1	0
B	1	0	1	0	1
C	0	1	0	1	0
D	1	0	1	0	1
E	0	1	0	1	0

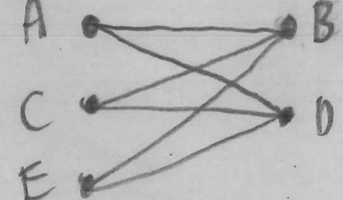
This means that every row corresponds to a vertex in G, every column corresponds to an edge in G, and the (i, j)-entry is 1 if and only if edge j is attached to vertex i.

(a) (3 points) Draw a picture of G. Is G bipartite?



Yes, G is bipartite.

Try to split into 2 sets:



(b) (5 points) Let v be the vertex of G that corresponds to the third row of the matrix above. How many length six paths are there from v to itself? Show your work.

(C)?

$$A^2 = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 2 & 0 & 2 \\ 0 & 3 & 0 & 3 & 0 \\ 2 & 0 & 2 & 0 & 2 \\ 0 & 3 & 0 & 3 & 0 \\ 2 & 0 & 2 & 0 & 2 \end{bmatrix}$$

← 3rd row

↑ 3rd row

A^3
A^4
A^5
A^6

There are 2 length 6 paths.

(c) (4 points) Does the graph G contain a Hamiltonian cycle? Justify your answer.

No, G does not have a Hamiltonian cycle. To have one, G would need 5 "workable edges". G has 6 total edges, and 2 "unworkable" ones, one of the 3 edges attaches to B and one of the 3 attached to D. So G has only 4 "workable" edges. Since G doesn't have enough "workable" edges to form a Hamiltonian cycle, it doesn't have one.

Extra scratch paper.