DO NOT OPEN THIS EXAM UNTIL YOU ARE INSTRUCTED TO DO SO!

Class: Math 61, Lecture 1 Instructor: Jonathan Rubin

Exam: Midterm I

Date: 21 October 2019

Time: 11:00 AM - 11:50 AM

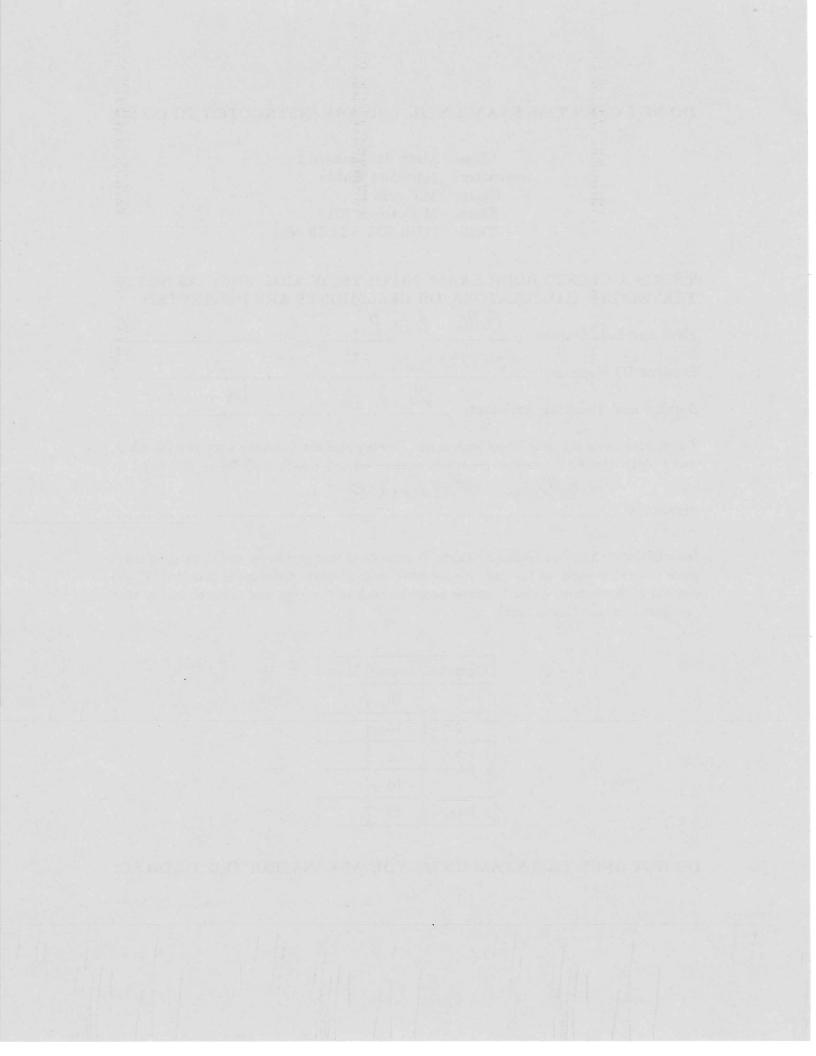
THIS IS A CLOSED BOOK EXAM. NO OUTSIDE AIDS, SUCH AS NOTES, TEXTBOOKS, CALCULATORS, OR CELLPHONES ARE PERMITTED.

First and Last Name:	
Student ID Number:	
Section and Teaching Assistant:	d Sonkup 1A
I understand that this is a closed book exam. I ce and I pledge that I have neither given nor receive	
Signature:	

Instructions: This is a 50-minute exam. It consists of four problems, and there is an extra piece of scratch paper at the end. Please write your answers in the space provided. If you run out of room, then please continue onto the back of the page and indicate clearly that you have done so. Good luck!

Question	Points	Score	
1	10		
2	10		
3	10		
4	10		
Total:	40		

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1. (10 points) Prove that the equation

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}$$

holds for every integer $n \geq 1$.

base
$$n=1$$
: $I(1n) = 1+1 \rightarrow \frac{1}{2} = \frac{1}{2}$

induction $n=n+1$: $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \cdots + \frac{1}{n(n)} + \frac{1}{n+1(n+2)} = \frac{n+1}{n+1}$

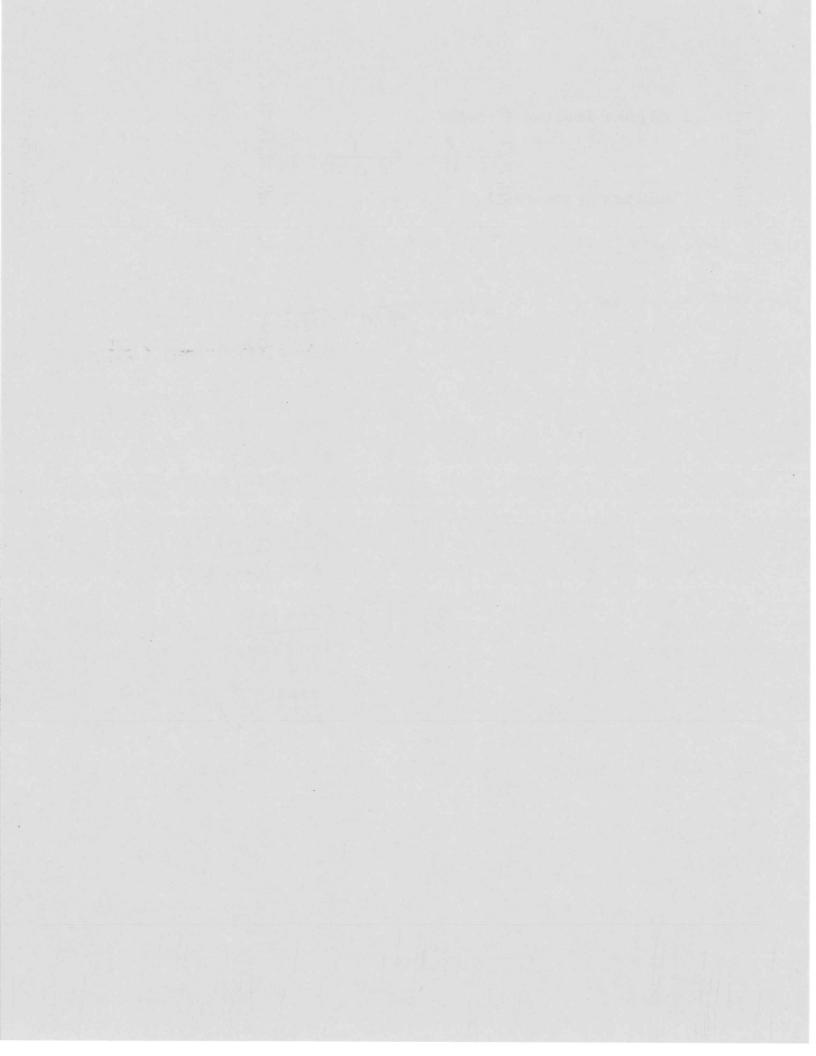
$$= \frac{n}{n+1} + \frac{1}{(n+1)(n+2)}$$

$$= \frac{n(n+2)}{(n+1)(n+2)} + \frac{1}{(n+1)(n+2)}$$

$$= \frac{n^2 + 2n + 1}{(n+1)(n+2)}$$

$$= \frac{(n+1)^2}{(n+1)(n+2)}$$

$$= \frac{n+1}{n+7}$$



2. (10 points) Consider the function

$$s(x) = \{x\} : \{a, b, c, d\} \to \mathcal{P}(\{a, b, c, d\}),$$

where $\mathcal{P}(\{a,b,c,d\})$ is the set of all subsets of $\{a,b,c,d\}$. This means that the function s sends a letter $x \in \{a,b,c,d\}$ to the *singleton set* $\{x\}$, whose only element is x.

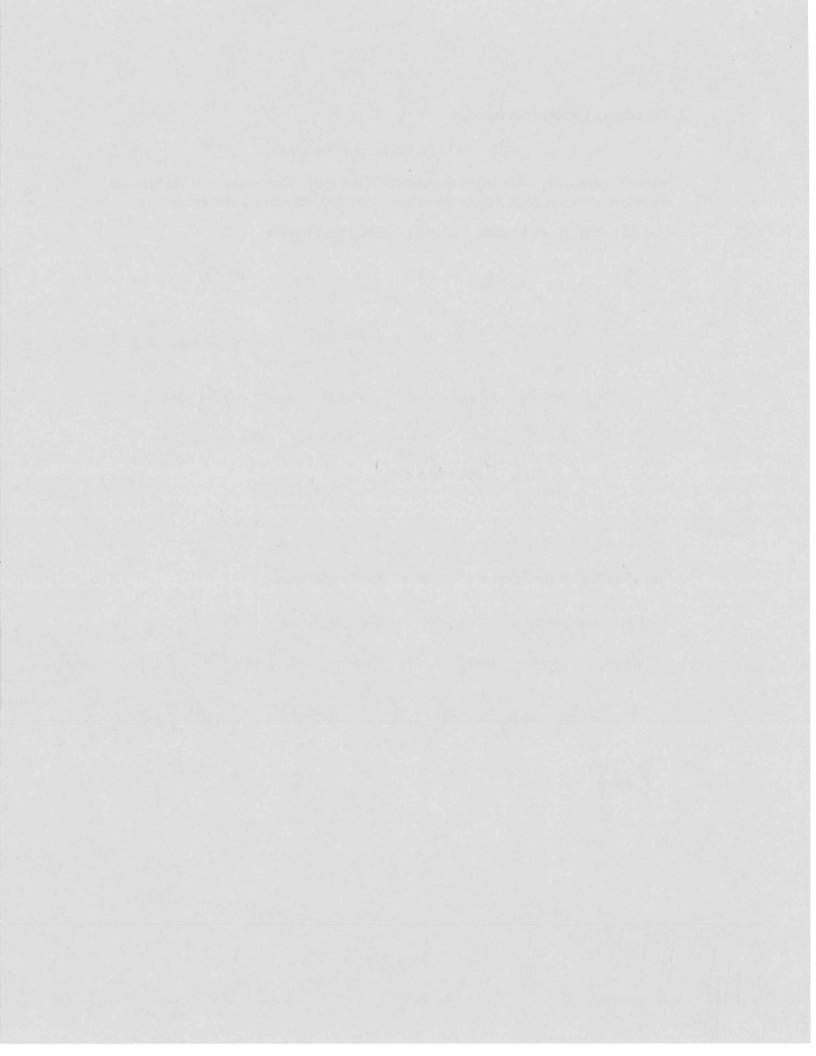
(a) (5 points) Is the function s injective? Justify your answer.

S is in jectilize as for any
$$x \in \{2, 6, c, d\}$$

 $S(x) = \{x\}$, Here fore any value $x \in \{2, 6, c, d\}$
Can only map to itself and therefore
it is the only unique value that maps
to itself (ore-to-ore).

(b) (5 points) Is the function s surjective? Justify your answer.

The function is not surjective as it does not map to every subset of \{a,b,c,d\},
it only maps to the subset \{a\}, \{a\}, \{c\}, \quad \{a\},
\{c\}, \quad \{a\},



3. (10 points) Consider the binary relation
$$R$$
 on $\{1, 2, 3, 4\}$ whose matrix is

This means that iRj is true if the (i, j)-entry is 1, and false if it is 0.

(a) (2 points) Is the relation R reflexive? Briefly explain.

The relation is not reflexive as not all
$$x \in \{1, 2, 3, 4\}$$
 set isfy $x R x$ in this matrix (example: $(4,4) \not\in R$).

(b) (2 points) Is the relation R symmetric? Briefly explain.

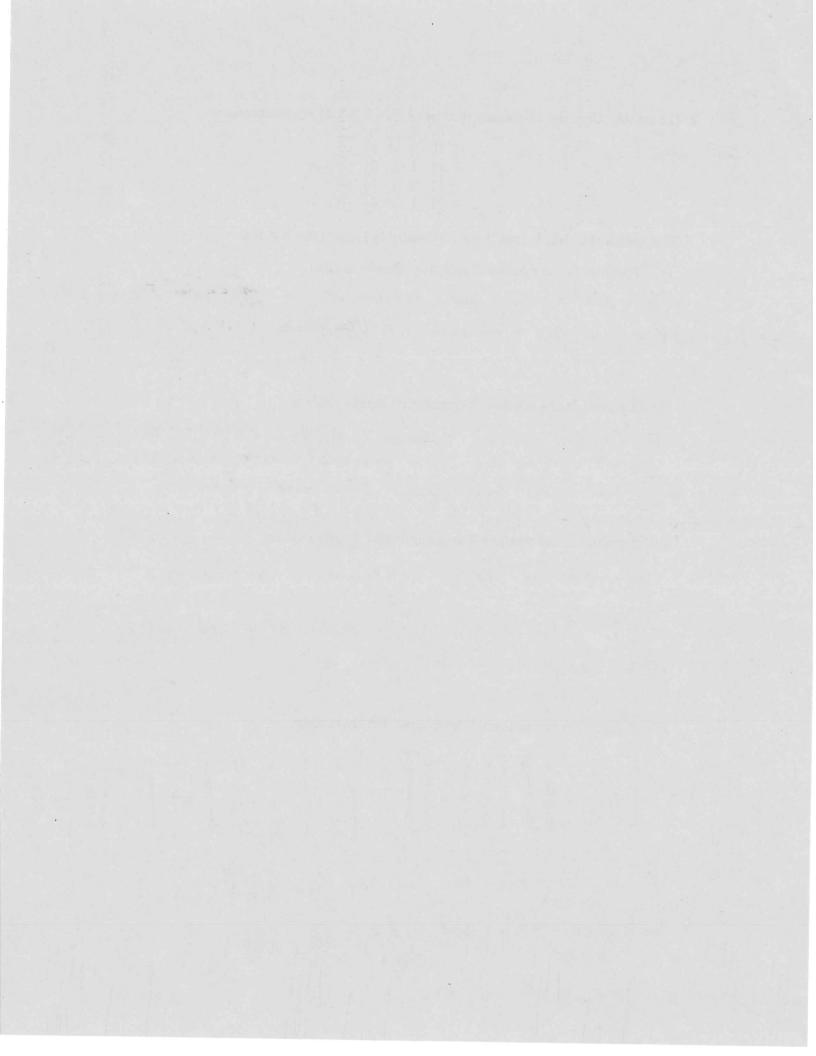
The relation is not symmetric as the matrix is not slong the main diagonal and far all x, y \(\xi_1, \xi_2, \xi_3, \xi_3, \xi_4 \xi_5 \) it is not always true that \(\xi_8 \xi_9 \) and \(\xi_8 \xi_1 \). For example, \((3, 1) \in R \), \(\lambda \text{int} \) (1,3) \(\xi_8 \). (c) (3 points) Is the relation R antisymmetric? Briefly explain.

the relation R is antisymmetric as for any X, Y & 21,2,3,43 such that XRy and yRx, then X=y.

(d) (3 points) Is the relation R transitive? Briefly explain.

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} = R$$

R is transitive as for any x, y, Z E \{1,2,3,43 such that xRy and yRZ, then xRZ.



- 4. (10 points) How many 5-letter (possibly nonsense) words w are there in $\{A, B, C, \ldots, Z\}$ that satisfy the following four properties?
 - (i) The word w starts with an A or B, and
 - (ii) the word w ends with an X, Y, or Z, and
 - (iii) the word w contains at least one C, and
 - (iv) the word w contains exactly one D.

Please simplify your answer as much as possible.

Zoption 3 option, at least one C, exactly are D

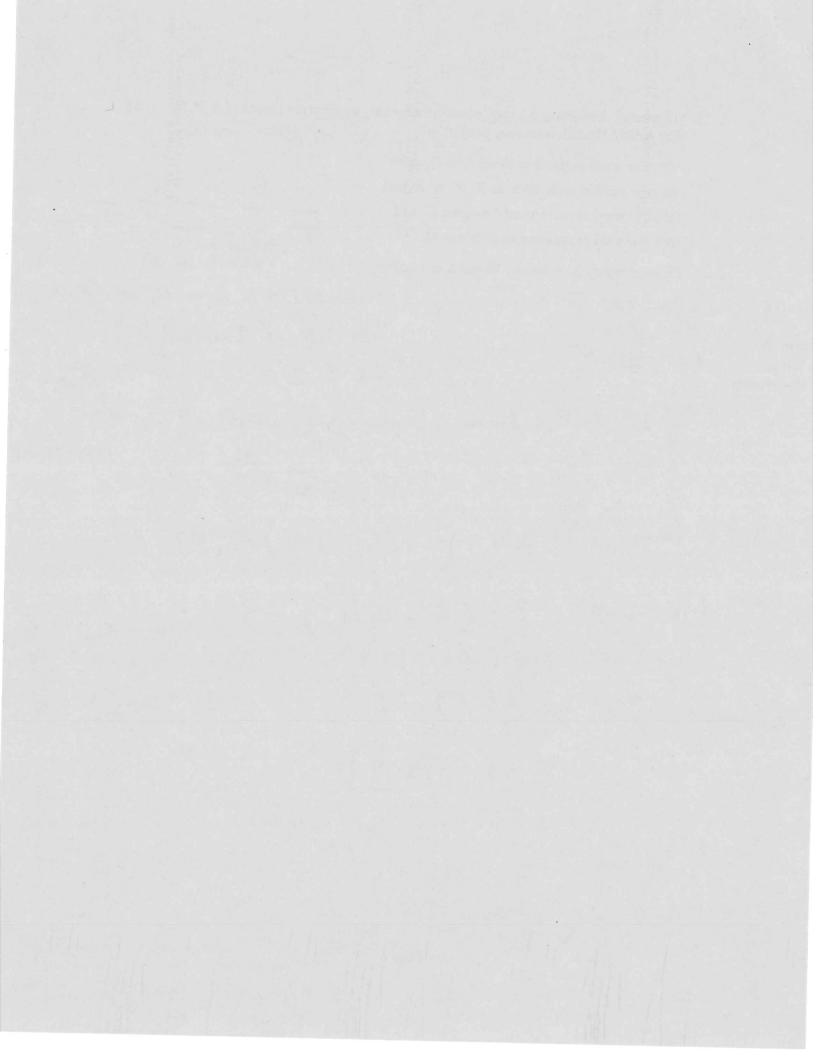
ACD, ADC

or ZC& 1D

punntation letter

{2C, 1 P, d 1 random: 1 option, 1 option, 2 le options = 3! · 26

{2C, 4D: 2 of 1& 1 option
$$\rightarrow \frac{3!}{2! \, 1!} = 3 \rightarrow \frac{200}{2! \, 1!} = 3 \rightarrow \frac{200}{2!} = 3 \rightarrow \frac{200}{2!$$



Extra scratch paper.

