# 19F-MATH61-1 Midterm I B

**TOTAL POINTS** 

### 39 / 40

#### **QUESTION 1**

## 1 Counting 9 / 10

- + 7 pts Mostly correct, but double-counted those with two C's and one D, or otherwise did not consider the difference between DCE and DCC.
- √ + 10 pts Complete, correct solution
- √ 1 pts Secondary counting mistake
- + 1 pts Got (i) right we're looking for things that satisfy all
- + 1 pts Got (ii) right we're looking for things that satisfy all
- + 1 pts Got (iii) right we're looking for things that satisfy all
- + 1 pts Got (iv) right we're looking for things that satisfy all
- + **0 pts** Blank / no answer / no progress towards a solution
- 1 by your count the last letter can't be C either

#### **QUESTION 2**

### Relations 10 pts

### 2.1 Symmetry 2 / 2

- √ + 1 pts Correct answer
- √ + 1 pts Good explanation
  - + **O pts** Incorrect
  - + 0 pts Insufficient explanation
  - + 0 pts Incorrect definition

## 2.2 Reflexivity 2/2

- √ + 1 pts Correct answer
- √ + 1 pts Good explanation
  - + 0 pts Insufficient explanation
  - + 0 pts Incorrect

### 2.3 Transitivity 3/3

- √ + 2 pts Correct answer
- √ + 1 pts Good explanation
  - + 0 pts Insufficient explanation
  - + 0 pts Incorrect
- + 2 pts Correct reasoning based on errors in matrix multiplication

# 2.4 Antisymmetry 3/3

- √ + 2 pts Correct answer
- √ + 1 pts Good explanation
  - + 0 pts Incorrect answer
  - + 0 pts Incorrect definition
  - + 0 pts Insufficient explanation
  - + 1 pts Correct definition

#### QUESTION 3

### Functions 10 pts

# 3.1 Surjectivity 5 / 5

- √ + 3 pts Correct answer
  - + 0 pts Incorrect answer
- √ + 2 pts Sufficient justification
  - + 1 pts Some justification
  - + **0 pts** Insufficient justification
- + 1 pts You know the definition of surjectivity, but have not applied it correctly.

### 3.2 Injectivity 5 / 5

- √ + 3 pts Correct answer
- √ + 2 pts Good justification
  - + 1 pts Some justification
  - + **0 pts** Insufficient or incorrect justification
  - + 0 pts Incorrect answer or nothing written
  - + 1 pts You know the definition of injectivity, but

have not applied it correctly.

#### **QUESTION 4**

# 4 Induction 10 / 10

- √ + 3 pts Correct base case.
- $\checkmark$  + 5 pts Correct argument in the induction step.
- $\checkmark$  + 2 pts Written in sentences, chains of equalities, good style.
- 1 pts You can't start the induction proof with the equality you are trying to prove. Write one long chain of equalities instead.
  - 1 pts Slight misunderstanding of induction

# DO NOT OPEN THIS TEST UNTIL YOU ARE INSTRUCTED TO DO SO!

Class: Math 61, Lecture 1

Instructor: Jonathan Rubin

Exam: Midterm I

Date: 21 October 2019

Time: 11:00 AM - 11:50 AM

THIS IS A CLOSED BOOK EXAM. NO OUTSIDE AIDS, SUCH AS NOTES, TEXTBOOKS, CALCULATORS. OR CELLPHONES ARE PERMITTED.

First and Last Name:

Student ID Number:

Section and Teaching Assistant:

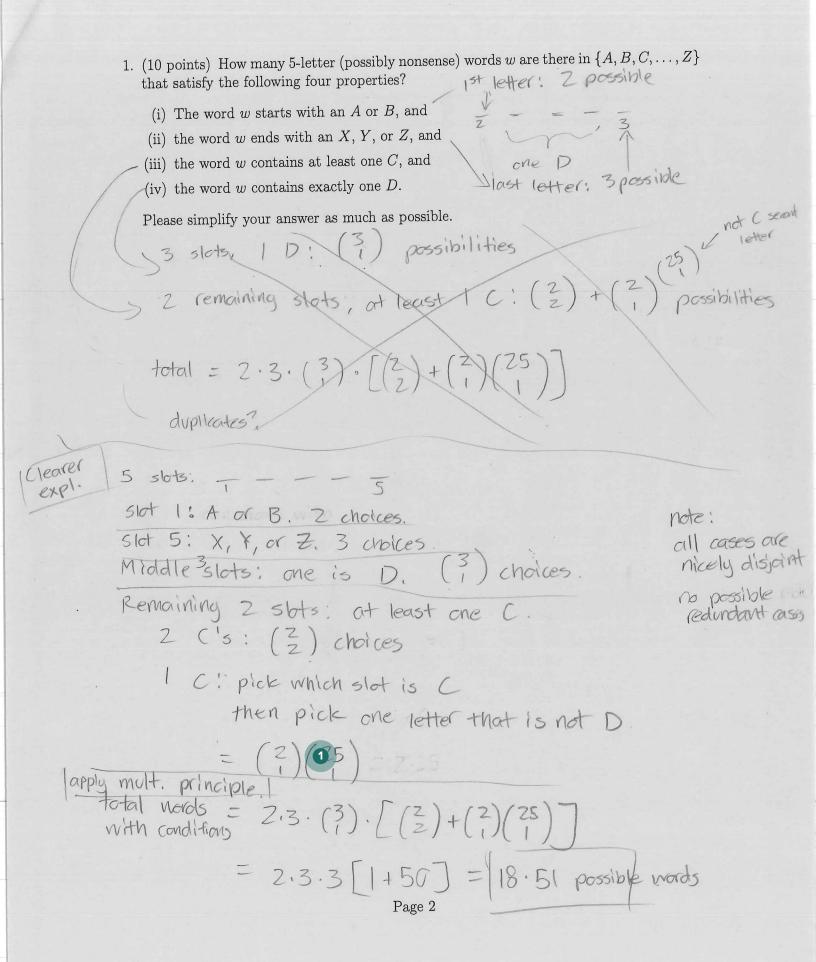
I understand that this is a closed book exam. I certify that the following work is mine alone, and I pledge that I have neither given nor received unauthorized assistance on this test.

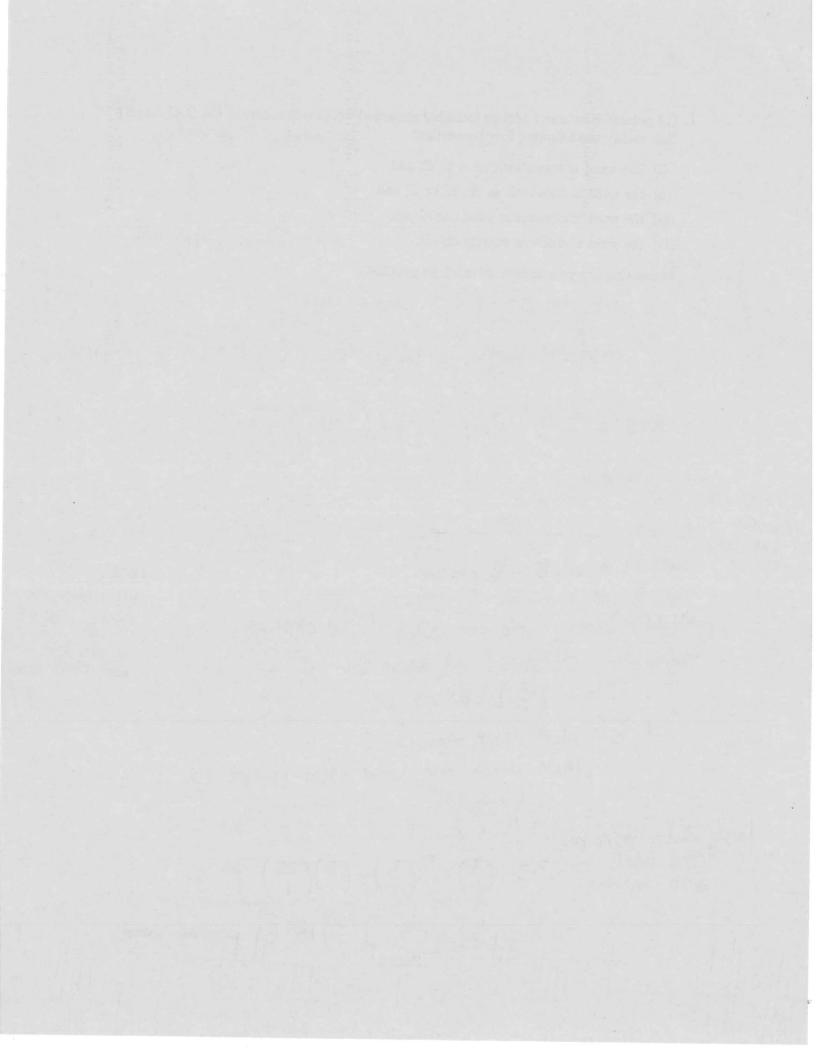
Signature:

Instructions: This is a 50-minute exam. It consists of four problems, and there is an extra piece of scratch paper at the end. Please write your answers in the space provided. If you run out of room, then please continue onto the back of the page and indicate clearly that you have done so. Good luck!

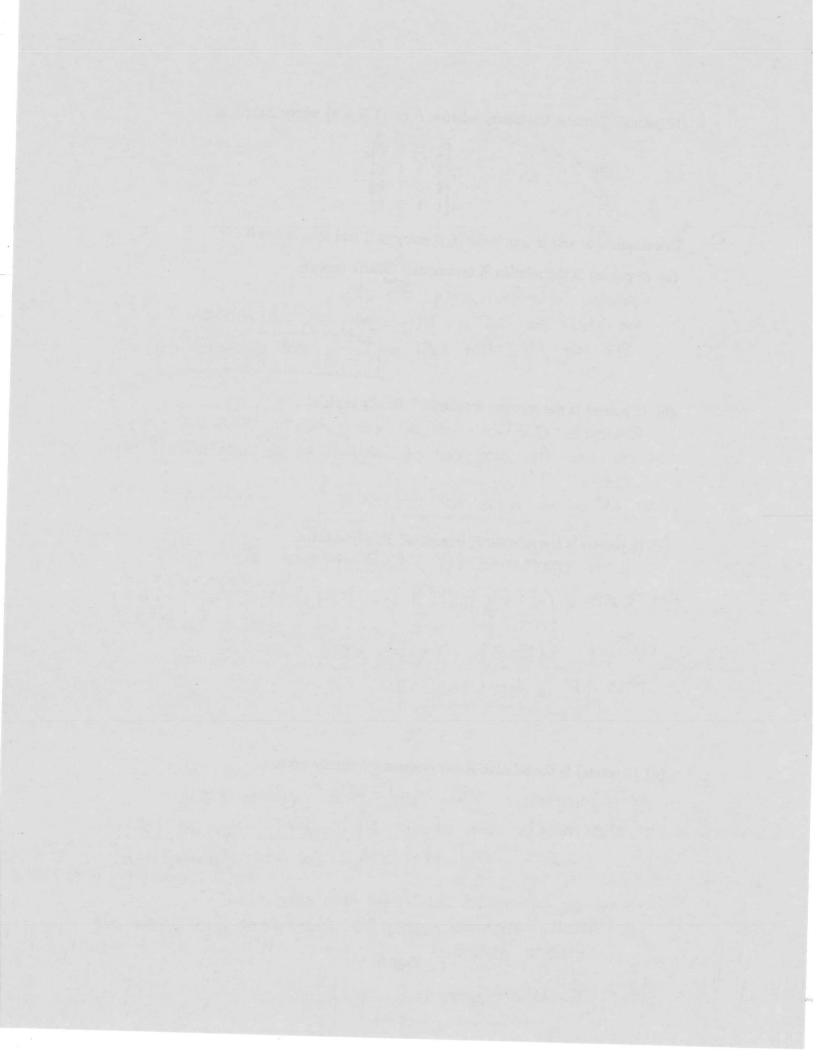
Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

DO NOT OPEN THIS TEST UNTIL YOU ARE INSTRUCTED TO DO SO!





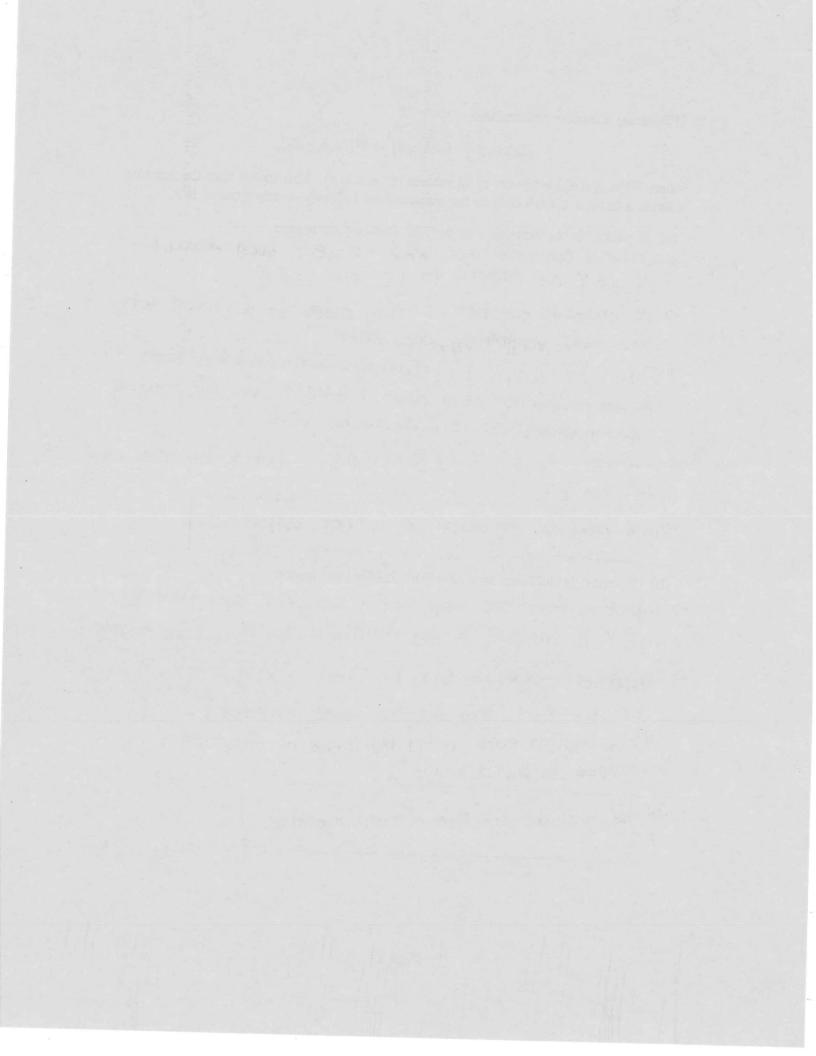
2.	(10 points) Consider the binary relation $R$ on $\{1, 2, 3, 4\}$ whose matrix is	
	(10 points) Consider the binary relation 12 of $(2,2,3,7)$ [10 points] Consider the binary relation 12 of $(2,2,3,7)$ [11 points] Relation 12 of $(2,2,3,7)$ [12 points] Relation 12 of $(2,2,3,7)$ [13 points] Relation 12 of $(2,2,3,7)$ [14 points] Relation 12 of $(2,2,3,7)$ [15 points] Relation 12 of $(2,2,3,7)$ [16 points] Relation 12 of $(2,2,3,7)$ [17 points] Relation 12 of $(2,2,3,7)$ [17 points] Relation 12 of $(2,2,3,7)$ [17 points] Relation 13 of $(2,2,3,7)$ [17 points] Relation 14 of $(2,2,3,7)$ [17 points] Relation 15 of $(2,2,3,7)$ [17 points] Relation 15 of $(2,2,3,7)$ [18 p	
	name 1 2 1 1 1 0	
	3 1 0 1 0	
This means that $iRj$ is true if the $(i, j)$ -entry is 1, and false if it is 0.		
	(a) (2 points) Is the relation R symmetric? Briefly explain.	
	Symmetric relation: xRy = yRx.	
	We check to see if Mij = Mij for all entires.	
	We check to see if Mij = Mji for all entries.  We see IRZ but ZRI so R is not symmetric	
	(b) (2 points) Is the relation R reflexive? Briefly explain.	
	Passaviva relation: for all x € 3 1/42/13, XIX.	
	-> We see the diagonal of matrix M for relation R is	
	-> 1R1, so  R is not reflexive	
	(c) (3 points) Is the relation $R$ transitive? Briefly explain.	
	R is transitive iff ROLL retines K.	
	ROP - MIM - (CCCC) (0000) - (0000) nonzeros (CCCC)	
	$R \circ R = M \circ M = \begin{pmatrix} c & c & c & c \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} c & c & c \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} c & c & c & c \\ 2 & 1 & 2 & c \\ 1 & 0 & 1 & c \\ 2 & 1 & 2 & c \end{pmatrix} \begin{pmatrix} c & c & c & c \\ 1 & 1 & 1 & c \\ 1 & 0 & 1 & c \\ 2 & 1 & 2 & c \end{pmatrix}$	
	We see x(ROR) y implies xRy	
	thus   R is transitive	
	D	
(d) (3 points) Is the relation R antisymmetric? Briefly explain.		
	Antisymmetric: xRy and yRx implies x=y	
	In the matrix, we check Mi, i & Mi, i for all i # i	
	and Mi,j.=Mj,i for i=j (always true; same entry in M)	
	> we see all entries do fulfill this condition	
	(Visially antices across the day from early attred are	
	(visually, entries across the diag from each other are opposite values) Page 3	
	=> R is antisymmetric	



3. (10 points) Consider the function 
$$s(x) = \{x\} : \{a,b,c,d\} \to \mathcal{P}(\{a,b,c,d\}),$$

where  $\mathcal{P}(\{a,b,c,d\})$  is the set of all subsets of  $\{a,b,c,d\}$ . This means that the function s sends a letter  $x \in \{a,b,c,d\}$  to the singleton set  $\{x\}$ , whose only element is x.

- (a) (5 points) Is the function s surjective? Justify your answer.
- all yey are mapped to by some xex.
- -> we observe function s only maps to singleton sets, i.e. sets containing one letter.
- -> Since P({a,b,c,d3}) contains non-singleton sets (sets containing more thom I letter), we find many counterexamples to surjectivity of s.
- -> counterexample: \(\xi\_a,b\)\(\frac{3}{3}\)\(\xi\)\(P(\xi\_a,b,c,d\)\(\frac{3}{3})\), but s does not mop to \(\xi\_a,b\)\(\xi\).
  - -> we conclude function s is not subjective /
    - (b) (5 points) Is the function s injective? Justify your answer.
  - -> Injective functions map x EX -> y EX such that no y EY is mapped to by multiple x EX (ane to one).
  - -> Suppose  $s(x_1) = s(x_2)$ . Then  $\frac{1}{2}x_1\frac{3}{3} = \frac{1}{2}x_2\frac{3}{3}$ , or  $x_1 = x_2$  (they are the same element). This implies each y in the range of function f is mapped to by a single x.
    - > we conclude function s is injective



4. (10 points) Prove that the equation

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}$$

holds for every integer  $n \geq 1$ .

1(1+1) = 1 holds, so the equation holds.

Then for the (n+1)th case, we hope to show 

LHS: we substitute our assumption to obtain

$$\frac{n}{n+1} + \frac{1}{(n+1)(n+2)}$$

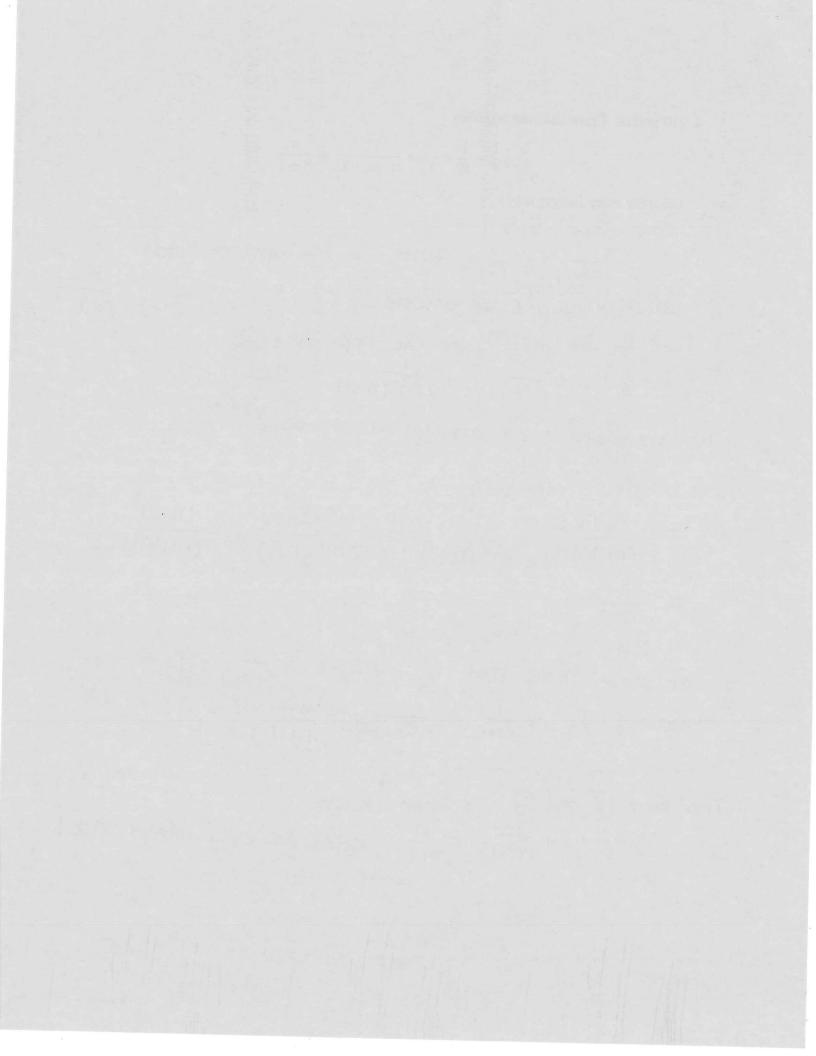
- $=\frac{n(n+2)}{(n+1)(n+2)}+\frac{1}{(n+1)(n+2)}=\frac{n^2+2n+1}{(n+1)(n+2)}=\frac{(n+1)^2}{(n+1)(n+2)}$

= RHS

So we nowe shown that if \frac{1}{2+\frac{1}{6}+\ldots\dots} = \frac{n}{n+1}

then  $\frac{1}{2} + \frac{1}{6} + \dots + \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+1+1)} = \frac{n+1}{(n+1)+1}$ 

This from () and (2), we have shown 12+1 + ... + in(n+1) = n holds for every integer nz1.



Extra scratch paper.

