Midterm 2 Version A

UCLA: Math 61, Winter 2018

Instructor: Jens Eberhardt
Date: 26 February 2017

- This exam has 4 questions, for a total of 34 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.

Name: STEVEN LA

ID number: [redacted]

Discussion section (please circle):

Day/TA	HUNT, CHRISTOPHER	HAN, KYUTAE	MENEZES, DEAN
Tuesday	1A	1C	1E
Thursday	1B	1D	1F

Question	Points	Score
1	12	12
2	8	8
3	6	6
4	8	7
Total:	34	33

20

13

34

Please note! The following two pages will not be graded. You must indicate your answers **here** *for them to be graded!*

Question 1.

Part	A	B	C	D
(a)				✓
(b)			✓	
(c)			✓	
(d)		✓		
(e)				✓
(f)			✓	

$$\sum_{i=0}^{n} 2^i = 1 + 2^1 + 2^2 + \cdots + 2^n = \frac{2^{n+1}-1}{\cancel{2-1}}$$

$n-(n-1)$ $\cancel{n-n}+1$

$n-(n-2)$ $\cancel{n-n}+2$

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (2 points) Let $a_n = a_{n-1} + 2^n$ and $a_0 = 1$. Then a_{100} equals

   A. $2^{100} + 1$
   B. $2^{101} + 1$
   C. $2^{100} - 1$
   D. $2^{101} - 1$

$100+1$: $2^{101}-1$

$n-(n-2)$
$\cancel{n-n}+2$
$n-(n-1)$
$\cancel{n-n}+1$

$k=n$:

$$a_n = a_{n-1} + 2^n \qquad a_{n-1} = a_{n-2} + 2^{n-1}$$
$$a_n = (a_{n-2} + 2^{n-1}) + 2^n \qquad a_{n-2} = a_{n-3} + 2^{n-2}$$
$$a_n = (a_{n-3} + 2^{n-2}) + 2^{n-1} + 2^n$$
$$\cdots$$
$$a_n = a_{n-k} + 2^{n-(k-1)} + 2^{n-(k-2)} + \cdots + 2^n$$
$$a_n = a_0 + 2^1 + 2^2 + \cdots + 2^n \Rightarrow \underbrace{1 + 2^1 + 2^2 + \cdots + 2^n}$$

$\cancel{1} + 2^1 + 2^2 + \cdots + 2^n$

$$\sum_{i=0}^{n} 2^i = \frac{2^{n+1}-1}{\cancel{2-1}}$$

$$a_n = 2^{n+1} - 1$$
$$a_{100} = 2^{100+1} - 1$$
$$2^{101} - 1$$

   (b) (2 points) The coefficient of $a^{10}b^{20}$ in the expansion of

$$(a+b)^{30}$$

   equals

   A. $C(30+10-1, 10-1)$
   B. $C(30+20-1, 20-1)$
   C. $C(30, 10)$
   D. $C(20, 10)$

$(a+b)^{n=30}$

$$\frac{30!}{20! \cdot 10!}$$

$$\sum_{k=0}^{30} C(30,k)\, a^{30-k} b^k \Rightarrow k = 20:$$

$$C(30,20)\, a^{\cancel{30-20}\,10} b^{20}$$

$$\frac{30!}{10!\ 20!} \qquad C(30,20) = C(30,10)$$

   (c) (2 points) Let X, Y be finite sets and $f: X \to Y$ a function. Under which conditions can you ensure that there are n distinct $x_1, x_2, \ldots, x_n \in X$, such that $f(x_1) = f(x_2) = \cdots = f(x_n)$.

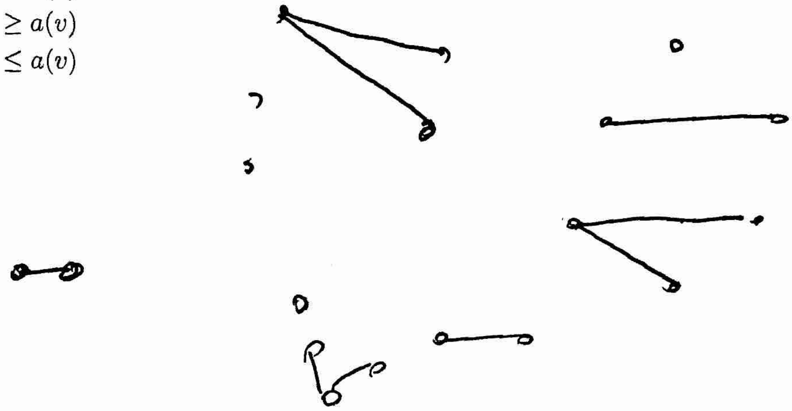
   A. $n|X| < |Y|$
   B. $n|X| > |Y|$
   C. $|X| > n|Y|$
   D. $|X| < n|Y|$

$|X| > n|Y|$

$|X| > n|Y|$ C

(d) (2 points) Let $G = (V, E)$ be a simple graph and $v \in V$ a vertex in G. Let $a(v)$ be the number of vertices adjacent to v and $\delta(v)$ the number of edges incident to v. Then

A. $\delta(v) > a(v)$
B. $\delta(v) = a(v)$
C. $\delta(v) \geq a(v)$
D. $\delta(v) \leq a(v)$



(e) (2 points) Which of the following is a linear homogeneous recurrence relation?

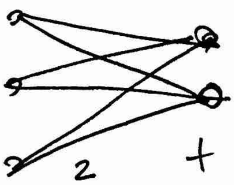
A. $a_n = 5a_{n-1} + na_{n-3}$
B. $a_n = a_{n-1} + 3a_0$
C. $a_n = a_{n-1}^2$
D. $a_n = 3(a_{n-1} + a_{n-3}) + 5a_{n-2}$

LHRR

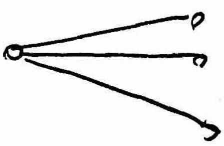
$a_n = 3a_{n-1} + 3a_{n-3} + 5a_{n-2}$ ✓

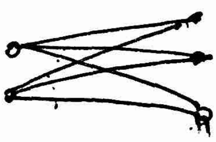
(f) (2 points) Let $G = K_{n,m}$ be the complete bipartite graph on n and m vertices. Then G has an Euler cycle if and only if

A. n and m are odd
B. $n + m$ is even
C. n and m are even
D. $n + m$ is odd

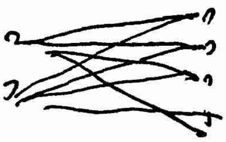


$n=1$ $m=3$ $2 + 3 = 5$

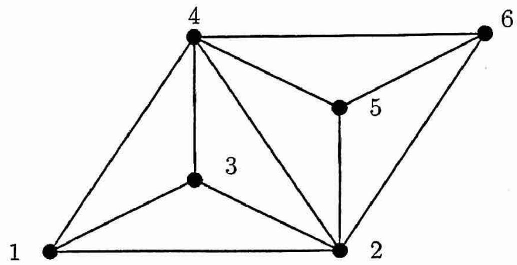




$n=2$ $n=4$

4  2 ✓

2. In the following questions, simply write down your answer. There is *no justification needed.* You can specify paths in simple graphs by a sequence of vertices.
Consider the following graph G.



(a) (2 points) Find a simple cycle in G with four edges containing 1 and 4.

Example : (1, 2, 3, 4, 1)

(b) (2 points) Is G bipartite?

No

(c) (2 points) Remove as many edges from the graph G as possible, such that the graph stays connected. How many edges are left in the end? (You are not allowed to remove vertices!)

5 edges

(d) (2 points) Let K_n be the complete graph on n vertices. How many edges does K_n have?

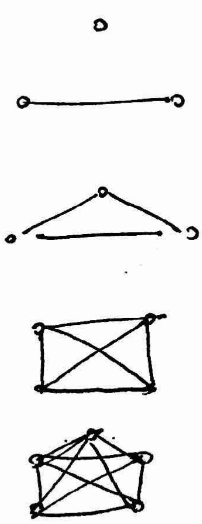
0 n=1

1 n=2

3 n=3

6 n=4

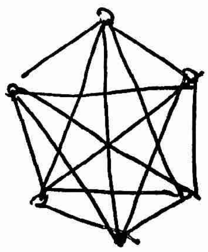
10 n=5



~~# edges of (K_n) = ($n-1$) + K_{n-1}, $K_0 = 0$~~

$$\# \text{edges} = \sum_{i=0}^{n-1} i$$

ok lol



n=6

15

$$= \boxed{\frac{n(n+1)}{2} - n}$$

3. Consider the following recurrence relation

$$a_n = -a_{n-1} + 2a_{n-2}$$

-1

2

with initial conditions

$$a_0 = 0, a_1 = 1.$$

Solve the recurrence relation in three steps.

(a) (2 points) Determine the characteristic polynomial and its roots.

Char. Poly: $x^2 - C_1 x - C_2$, $C_1 = -1$, $C_2 = 2$

$x^2 - (-1)x - 2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

-2	1
$2 \cdot -1$	$2 - 1$

$\boxed{\text{Roots: } -2, 1}$

✓

(b) (2 points) Determine the general solution.

Since we have 2 distinct real roots,

Gen. sol:

$S_n = a(-2)^n + b(1)^n$ * $[(1)^n = 1 \ \forall n \in \mathbb{R}]$ ✓

$\Rightarrow \boxed{S_n = a(-2)^n + b}$, $a, b \in \mathbb{R}$

$-2a - a$

$-3a$

(c) (2 points) Determine the solution fulfilling the initial conditions.

$S_0 = a_0 = 0 = a(-2)^0 + b$

$0 = a + b$

$S_1 = a_1 = 1 = a(-2)^1 + b$

$1 = -2a + b$

$e \quad 0 = a + b$

$1 = -3a$

$a = \frac{-1}{3}$

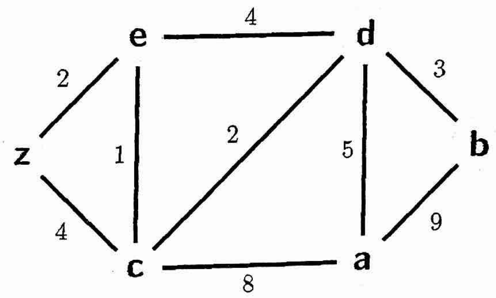
$\frac{-1}{3} + b = 0 \Rightarrow b = \frac{1}{3}$

$\Rightarrow S_n = \left(\frac{-1}{3}\right)(-2)^n + \frac{1}{3}$

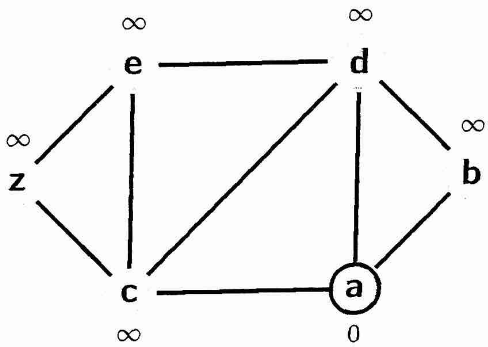
$\Rightarrow \boxed{a_n = \left(\frac{-1}{3}\right)(-2)^n + \frac{1}{3}}$

✓

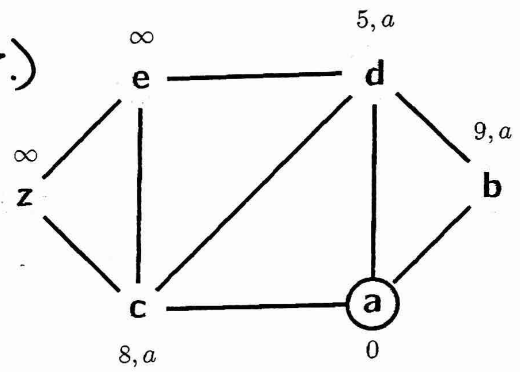
4. (8 points) Apply the next two iterations of Dijkstra's algorithm to find the shortest path from a to z in the following graph. ~~In each step~~, annotate each vertex x with $L(x)$ and $P(x)$, as shown. Circle the vertices already visited. Use the provided blank graphs. If you make a mistake, clearly cross it out and continue using the next blank graph.



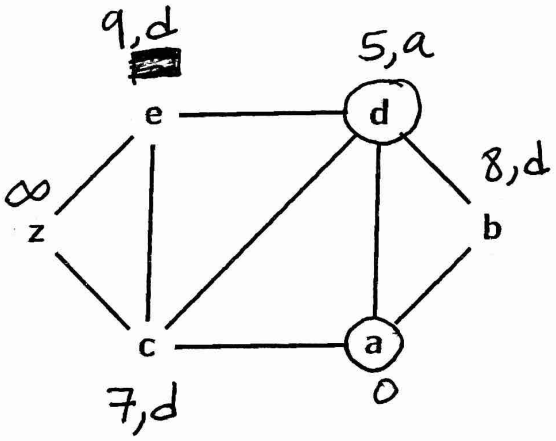
I.)



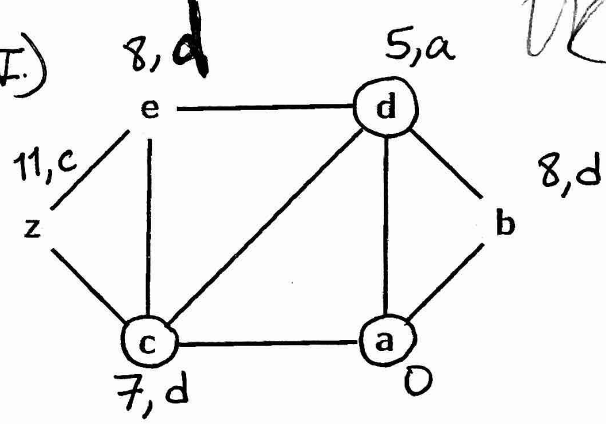
II.)



III.)



IV.)



OK