Midterm 1 (Version B)

UCLA: Math 61, Winter 2018

Instructor: Jens Eberhardt
Date: 02 February 2017

- This exam has 4 questions, for a total of 36 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.

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Discussion section (please circle):

Day/TA	HUNT, CHRISTOPHER	HAN, KYUTAE	MENEZES, DEAN
Tuesday	1A	1C	1E
Thursday	1B	1D	1F

Question	Points	Score
1	12	10
2	6	6
3	10	10
4	8	8
Total:	36	34

Please note! The following two pages will not be graded. You must indicate your answers here for them to be graded!

Question 1.

Part	A	B	C	D
(a)		✓		
(b)				✓
(c)	✓			
(d)			✓	
(e)	✓			
(f)	✓			

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (2 points) Let $X = \{0, 1, 2, 3\}$. For a set Y denote by $\mathcal{P}(Y) = \{S | S \text{ is a subset of } Y\}$ the power set of X. Then

      A. $|\mathcal{P}(X \times X)| = 2 \cdot 16$
      B. $|\mathcal{P}(X \times X)| = 2^{16}$
      C. $|\mathcal{P}(X \times X)| = 2 \cdot 8$
      D. $|\mathcal{P}(X \times X)| = 2^8$

   (b) (2 points) Let $n \geq 1$ be a positive integer. Then

$$\prod_{i=1}^{n} 3i = 3 \cdot 6 \cdot 9$$

3, 18, 162, 1944

$3^4 \cdot 4!$

$3^4 = 81$

   is equal to

      A. $(3n)!$
      B. $3^n \frac{n(n+1)}{2}$
      C. $\frac{3^n 3^{n+1}}{2}$
      D. $3^n \cdot n!$

   (c) (2 points) Define a partition $\mathcal{P}$ on $\{0, 1, 2, 3\}$ by

$$\mathcal{P} = \{\{0\}, \{1, 3\}, \{2\}\}.$$

   Let $R_{\mathcal{P}}$ be the associated equivalence relation on $\{0, 1, 2, 3\}$. Then

      A. $R_{\mathcal{P}} = \{(0, 0), (1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$
      B. $R_{\mathcal{P}} = \{(0, 0), (1, 3), (3, 1), (2, 2)\}$
      C. $R_{\mathcal{P}} = \{\{0\}, \{1, 3\}, \{2\}\}$
      D. $R_{\mathcal{P}} = \{0, 1, 2, 3\}$

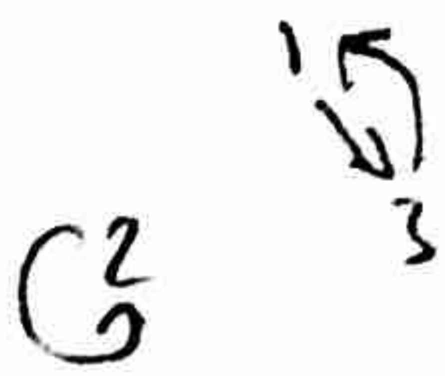
(d) (2 points) Let $X = \{1, 2, 3\}$, then $R = \{(1,3), (2,2), (3,1)\}$ is

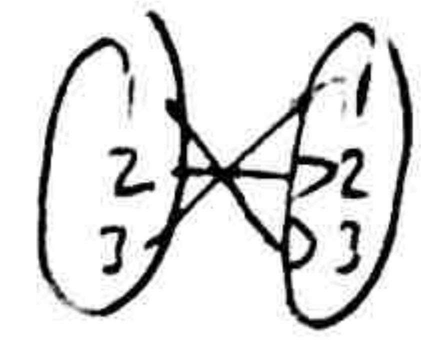
A. not symmetric

B. reflexive

C. bijective

D. antisymmetric





(e) (2 points) Let $X = \{a, b\}$. Denote by $X^{\geq 3}$ the set of all strings over X of length bigger or equal than three and by X^* the set of all strings over X. Then

A. $|X^* - X^{\geq 3}| = 5 = |X^{\leq 2}|$ a, b, ab, ba, λ

B. $|X^* - X^{\geq 3}| = 6$

C. $|X^* - X^{\geq 3}| = 7$

D. $|X^* - X^{\geq 3}| = 8$

(f) (2 points) Let $X = \mathbb{Z}_{\geq 0} = \{0, 1, 2, \dots\}$. Define an equivalence relation R on X by:

$$xRy \text{ if } x - y \text{ is divisible by } 3.$$

Then the partition $\mathcal{P}_R$ associated to the relation R is:

A. $\mathcal{P}_R = \{\{0, 3, 6, \dots\}, \{1, 4, 7, \dots\}, \{2, 5, 8, \dots\}\}$

B. $\mathcal{P}_R = \{\{0, 2, 4, \dots\}, \{1, 3, 5, \dots\}\}$

C. $\mathcal{P}_R = \{\{0\}, \{1\}, \{2\}, \dots\}$

D. $\mathcal{P}_R = \{(0,0), (0,3), (3,0) \dots, (3,3), (3,6), (6,3), \dots,$
$(1,1), (1,4), (4,1), \dots, (4,4), (4,7)(7,4), \dots,$
$(2,2), (2,5), (5,2), \dots, (5,5), (5,8)(8,5) \dots\}$

2. (6 points) Prove by induction that

$$\sum_{i=1}^{n}(2i-n)=n$$

for any integer $n \geq 1$.
Hint: Use

$$\sum_{i=1}^{n+1}(2i-(n+1))=\left(\sum_{i=1}^{n+1}(2i-n)\right)-(n+1).$$

Base case: $n=1$

$$\sum_{i=1}^{1}(2i-n)=1=2(1)-1=1 \quad //$$

✓

Inductive step: Assume statement true for n, prove statement is true for $n+1$.

$$\sum_{i=1}^{n+1}(2i-(n+1))=\left(\sum_{i=1}^{n+1}(2i-n)\right)-(n+1)$$

$$=\left(\sum_{i=1}^{n}(2i-n)\right)+(2(n+1)-n)-(n+1)$$

$$=n+(2(n+1)-n)-(n+1)$$

$$=2n+2-n-1$$

$$=n+1 \quad //$$

✓

Thus, $\sum_{i=1}^{n}(2i-n)=n$ by mathematical induction. ■

6

3. In the following questions, simply write down your answer. There is *no justification needed.* Do not simplify expressions as $2^4, 6!, C(n,r), \ldots$.

(a) (2 points) Let $X = \{1,2,3,4,5,6\}$. Determine the number of elements of the following set

$$\{S \mid S \text{ is a subset of } X \text{ and } |S| = 4\}.$$

C(6,4)

(b) (2 points) Determine the number of 5-bit strings starting in 101.

1 0 1 _ _
2 2

2^2

(c) (2 points) Determine the number of 5-bit strings ending in 010.

_ _ 0 1 0
2 2

2^2

(d) (2 points) Determine the number of 5-bit strings starting in 101 or ending in 010.

$2^2 + 2^2$

$\boxed{2^3}$

(e) (2 points) You have three friends: Rocco, Gina and Hans. And you have seven different sweets: a popsicle, a piece of apple pie, a chocolate bar, a Berliner, a jelly doughnut, a marshmallow and a lemon drop.
You want to give two sweets to Rocco, three to Gina and two to Hans. In how many ways could you do this?

R R G G G H H

$\frac{7!}{2!\,3!\,2!}$

4. Answer the following questions, *justifying* your answers. (If an answer is Yes, explain why. If an answer is No, give a counterexample.)

(a) (2 points) Let $X = \{a, b, c, \ldots, z\}$ be the alphabet. Let α and β be strings over X. Is

$$\alpha\beta = \beta\alpha?$$

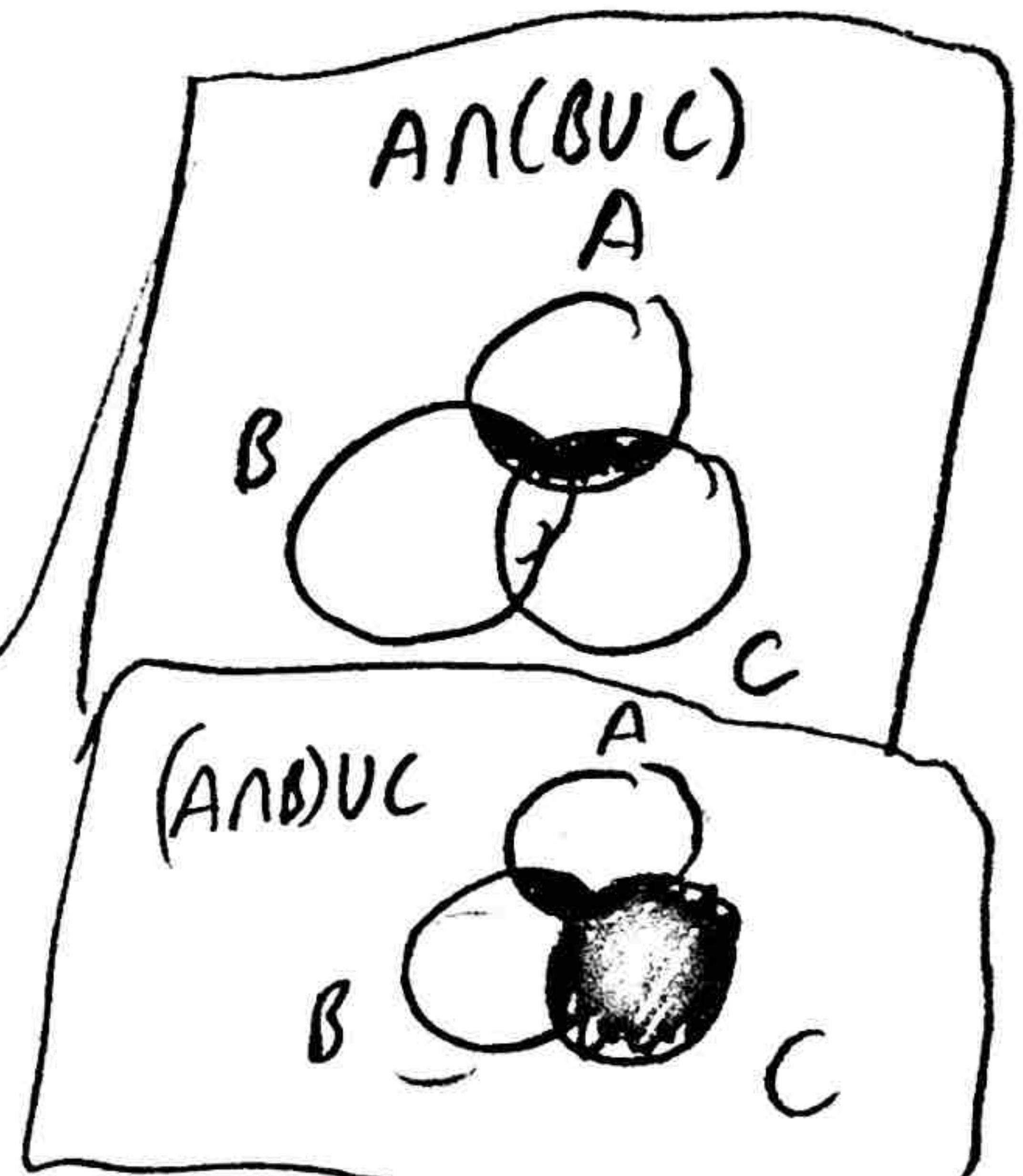
No, let $\alpha = a$, $\beta = b$

$(\alpha\beta = ab) \neq (\beta\alpha = ba)$

(b) (2 points) Let A, B, C be sets. Then

$$A \cap (B \cup C) = (A \cap B) \cup C.$$

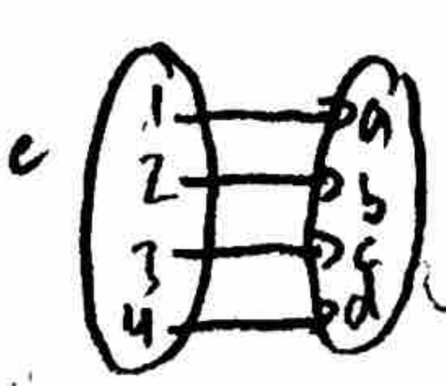
No, the diagrams illustrate they are different



(c) (2 points) Let $X = \{1, 2, 3, 4\}$ and $Y = \{a, b, c, d\}$. Is there a bijective function $f : X \to Y$?

Yes, let $F: X \to Y$ be represented by the following arrow diagram

because the function is surjective and injective, it is bijective



Each $x \in X$ maps to a distinct $y \in Y$ such that if $f(x) = f(y) \Rightarrow x = y$. The function is <u>injective</u>.

Each $y \in Y$ maps to an $x \in X$ such that for any $y \in Y$ there exists an $f(x) = y$. The function is <u>surjective.</u>

(d) (2 points) Define $f : \mathbb{R} \to \mathbb{R}_{\geq 0}, x \mapsto x^2$. Is f surjective?

Yes, for all $y \in Y$ there exists an $x \in X$ such that $f(x) = y$.

Let $y = x^2$, $x = \pm\sqrt{y}$ $f(\sqrt{y}) = (\sqrt{y})^2 = y$

$f(-\sqrt{y}) = (\sqrt{y})^2 = y$