Midterm 2 Version B

UCLA: Math 61, Winter 2018

Instructor: Jens Eberhardt
Date: 26 February 2017

- This exam has 4 questions, for a total of 34 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.

Name: ______________________________

ID number: ______________________________

Discussion section (please circle):

Day/TA	HUNT, CHRISTOPHER	HAN, KYUTAE	MENEZES, DEAN
Tuesday	1A	1C	1E
Thursday	1B	1D	1F

Question	Points	Score
1	12	10
2	6	6
3	8	6
4	8	8
Total:	34	30

-4

Please note! The following two pages will not be graded. You must indicate your answers here for them to be graded!

Question 1.

Part	A	B	C	D
(a)		X		
(b)		X		
(c)		X		
(d)				X
~~(e)~~			X	
(f)	X			

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

(a) (2 points) The coefficient of $a^{10}b^{20}$ in the expansion of

$$(a+b)^{30}$$

$k=20$
$n=30$

$n-k=30-20=10$

equals

A. $C(30+\cancel{10}-1, 10-1)$
B. $C(30,10)$
C. $C(20,10)$
D. $C(30+\cancel{20}-1, 20-1)$

$C(30, 20)$

(b) (2 points) Let $a_n = a_{n-1} + 2^n$ and $a_0 = 1$. Then a_{100} equals

A. $2^{100}+1$
B. $2^{101}-1$
C. $2^{101}+1$
D. $2^{100}-1$

$a_n = a_{n-1} + 2^n$
$= (a_{n-2} + 2^{n-1}) + 2^n$
$= a_{n-2} + 2^{n-1} + 2^n$
$= (a_{n-3} + 2^{n-2}) + 2^{n-1} + 2^n$
$= a_{n-3} + 2^{n-2} + 2^{n-1} + 2^n$

$2^{100} + a_{99}$

For $k=n$

$= a_{n-k} + \sum_{i=n-(k-1)}^{n} 2^i = a_{n-k} + \sum_{i=n-k+1}^{n} 2^i = a_0 + \sum_{i=1}^{n} 2^i$

$n-k+1$

$= 1 + \left(\frac{2^{n+1}-1}{2-1} - 1\right)$

$2^{101}-1$

(c) (2 points) Which of the following is a linear homogeneous recurrence relation?

A. $a_n = 5a_{n-\cancel{1}} + na_{n-3}$
B. $a_n = 3(a_{n-1} + a_{n-3}) + 5a_{n-2} = 3a_{n-1} + 3a_{n-3} + 5a_{n-2}$
C. $a_n = a_{n-1} + 3a_0$
D. $a_n = \cancel{a^2_{n-1}}$

(d) (2 points) Let $G = K_{n,m}$ be the complete bipartite graph on n and m vertices. Then G has an Euler cycle if and only if

A. n and $\cancel{m}$ are odd

B. $n + m$ is even

C. $n + m$ ~~is~~ odd

D. n and m are even

4 3

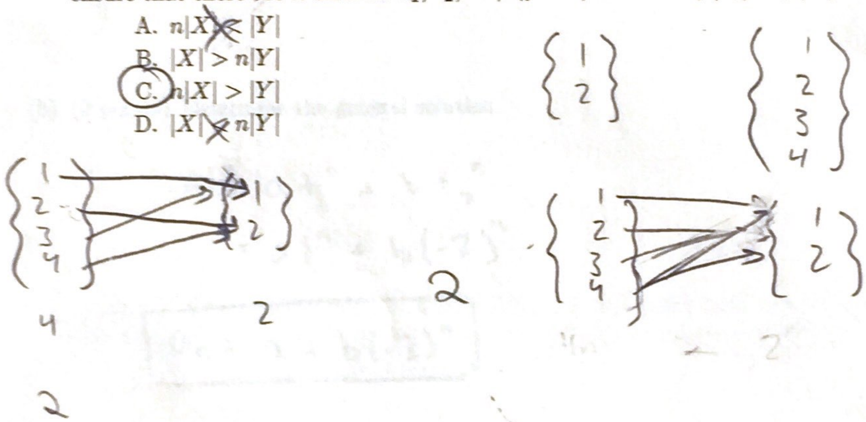
(e) (2 points) Let X, Y be finite sets and $f : X \to Y$ a function. Under which conditions can you ensure that there are n distinct $x_1, x_2, \ldots, x_n \in X$, such that $f(x_1) = f(x_2) = \cdots = f(x_n)$.

A. $n|X| \cancel{<} |Y|$

B. $|X| > n|Y|$

C. $n|X| > |Y|$

D. $|X| \cancel{<} n|Y|$



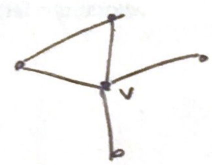
(f) (2 points) Let $G = (V, E)$ be a simple graph and $v \in V$ a vertex in G. Let $a(v)$ be the number of vertices adjacent to v and $\delta(v)$ the number of edges incident to v. Then

A. $\delta(v) = a(v)$

B. $\delta(v) \cancel{>} a(v)$

C. $\delta(v) \geq a(v)$

D. $\delta(v) \leq a(v)$



$a(v) = 4$ $\delta(v) = 4$

2. Consider the following recurrence relation

$$a_n = -a_{n-1} + 2a_{n-2}$$

with initial conditions

$$a_0 = 0, a_1 = 1.$$

Solve the recurrence relation in three steps.

(a) (2 points) Determine the characteristic polynomial and its roots.

$t^2 + t - 2 = 0$ $\frac{-2}{-1, 2}$

$(t-1)(t+2) = 0$

$\boxed{\text{Roots: } t_1 = 1,\ t_2 = -2}$

(b) (2 points) Determine the general solution.

$a_n = a t_1^n + b t_2^n$

$= a 1^n + b(-2)^n$

$\boxed{a_n = a + b(-2)^n}$

(c) (2 points) Determine the solution fulfilling the initial conditions.

$a_0 = 0$ $a_1 = 1$

$0 = a + b(-2)^0 = a + b$

$1 = a + b(-2)^1 = a - 2b$

$a = -b$

$1 = (-b) - 2b$

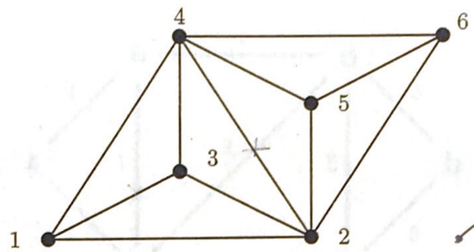
$1 = -3b,\ b = -\frac{1}{3}$

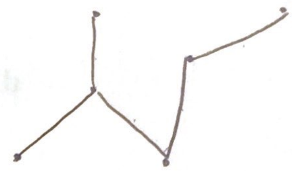
$\downarrow$

$a = \frac{1}{3}$

$\boxed{a_n = \frac{1}{3} - \frac{1}{3}(-2)^n}$

3. In the following questions, simply write down your answer. There is *no justification needed.* You can specify paths in simple graphs by a sequence of vertices.
Consider the following graph G.





(a) (2 points) Find a simple cycle in G with four edges containing 1 and 4.

$\boxed{(1,4,3,2,1)}$

(b) (2 points) Is G bipartite?

$V_1 = \{3, 5 \cdots -$

$V_2 = \{2, 4 - \cdots$

$\boxed{\text{No}}$

(c) (2 points) Remove as many edges from the graph G as possible, such that the graph stays connected. How many edges are left in the end? (You are not allowed to remove vertices!)

$\boxed{5 \text{ edges}}$

(d) (2 points) Let K_n be the complete graph on n vertices. How many edges does K_n have?

0 $K_1 \to$ 1 3 6 10 15

$n=$ 1 2 3 $2^3=8$ 4 $2^4=16$ 5

$2^1=2$ $2^2=4$

$K_n = K_{n-1} + (n-1) = K_{n-1} + n - 1$

$= (K_{n-2} + (n-1) - 1) - 1$

$= K_{n-2} + n - 3$

$= (K_{n-3} + (n-2) - 1) - 3$

$= K_{n-3} + n - 6$

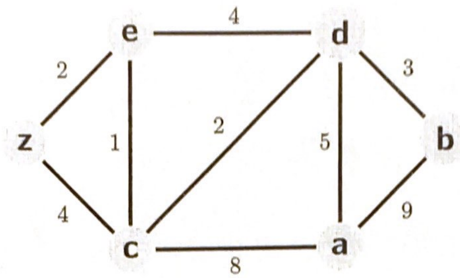
$= (K_{n-4} + (n-3) - 1) - 6$

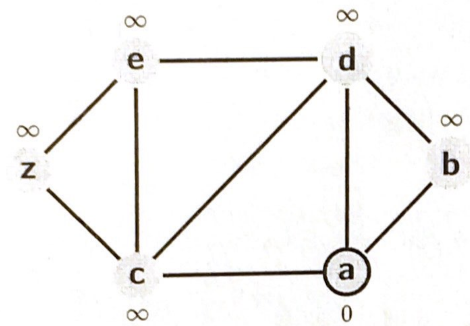
$= K_{n-4} + n - 10$

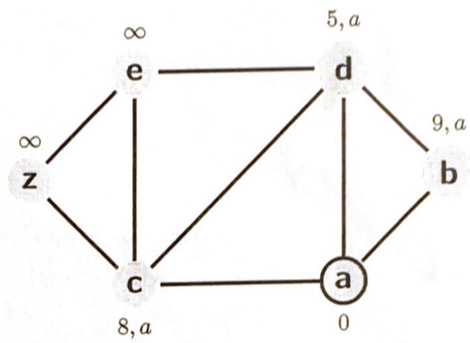
$\boxed{K_n \text{ has } K_{n-1} + (n-1) \text{ edges}}$

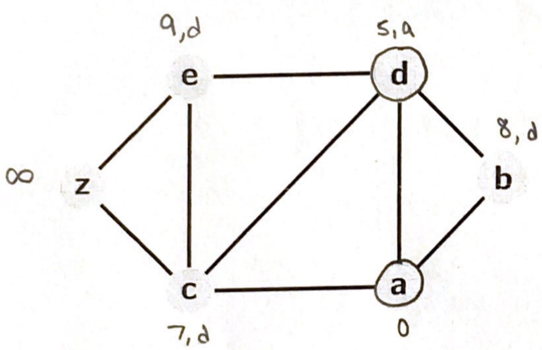
closed form...

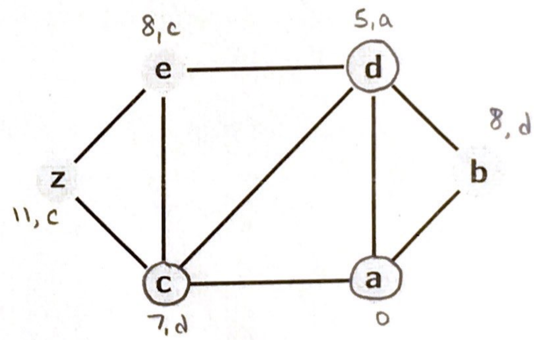
4. (8 points) Apply the next **two** iterations of Dijkstra's algorithm to find the shortest path from a to z in the following graph. In each step, annotate each vertex x with $L(x)$ and $P(x)$, as shown. Circle the vertices already visited. Use the provided blank graphs. If you make a mistake, clearly cross it out and continue using the next blank graph.

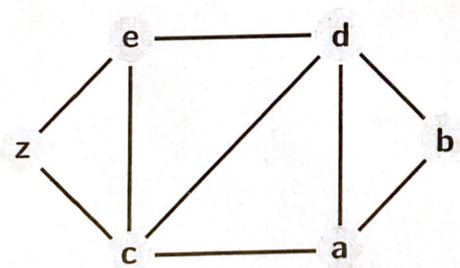


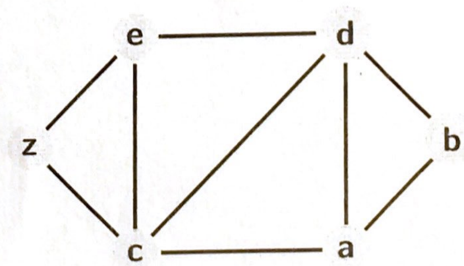












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