

# Math 61 Midterm 2

Ryan Kourosh Riahi

TOTAL POINTS

**74 / 100**

QUESTION 1

1 10 / 10

- ✓ + 2 pts a) False
- ✓ + 2 pts b) False
- ✓ + 2 pts c) False
- ✓ + 2 pts d) True
- ✓ + 2 pts e) True

QUESTION 2

30 pts

2.1 a 6 / 6

- ✓ - 0 pts Correct

2.2 b 4 / 4

- ✓ - 0 pts Correct (based on answer to a)

2.3 c 10 / 10

- ✓ - 0 pts Correct

2.4 d 10 / 10

- ✓ + 10 pts Correct
- + 2 pts correct constant on  $(-1)^n$
- + 2 pts correct constant on  $5^n$
- + 6 pts  $g(n)=-2$
- + 2 pts  $g(n)=c$
- 2 pts Minor computational error
- + 0 pts No significant progress

QUESTION 3

3 19 / 20

- ✓ - 1 pts Gave the length of the shortest path from a to d as the final answer

QUESTION 4

25 pts

4.1 a 0 / 10

+ 10 pts Correct

- ✓ + 0 pts Did not draw a Hamiltonian cycle

4.2 b 0 / 15

+ 15 pts Correct

- ✓ + 0 pts Solution lacked any significant content/tried to use counting method to conclude that no path exists (and believed it worked/used it improperly)

- 8 pts Solution is vaguely worded, incomplete or includes false claims that doesn't affect final argument.

- 3 pts Does not explain why certain edges must be used/excluded in a potential Hamiltonian cycle.

QUESTION 5

15 pts

5.1 a 10 / 10

- ✓ - 0 pts Bin 1: Correct (ignoring wording issues that do not create an issue in the argument)

5.2 b 5 / 5

- ✓ - 0 pts Correct

# MIDTERM 2 (MATH 61)

FRIDAY, MAY 17TH

Name: Ryan Riaki

ID: 105138860

Circle your discussion section:

Tuesday      Thursday

2A

2B

TA: Harris Khan

2C

2D

TA: Fred Vu

2E

2F

TA: Matthew Stone

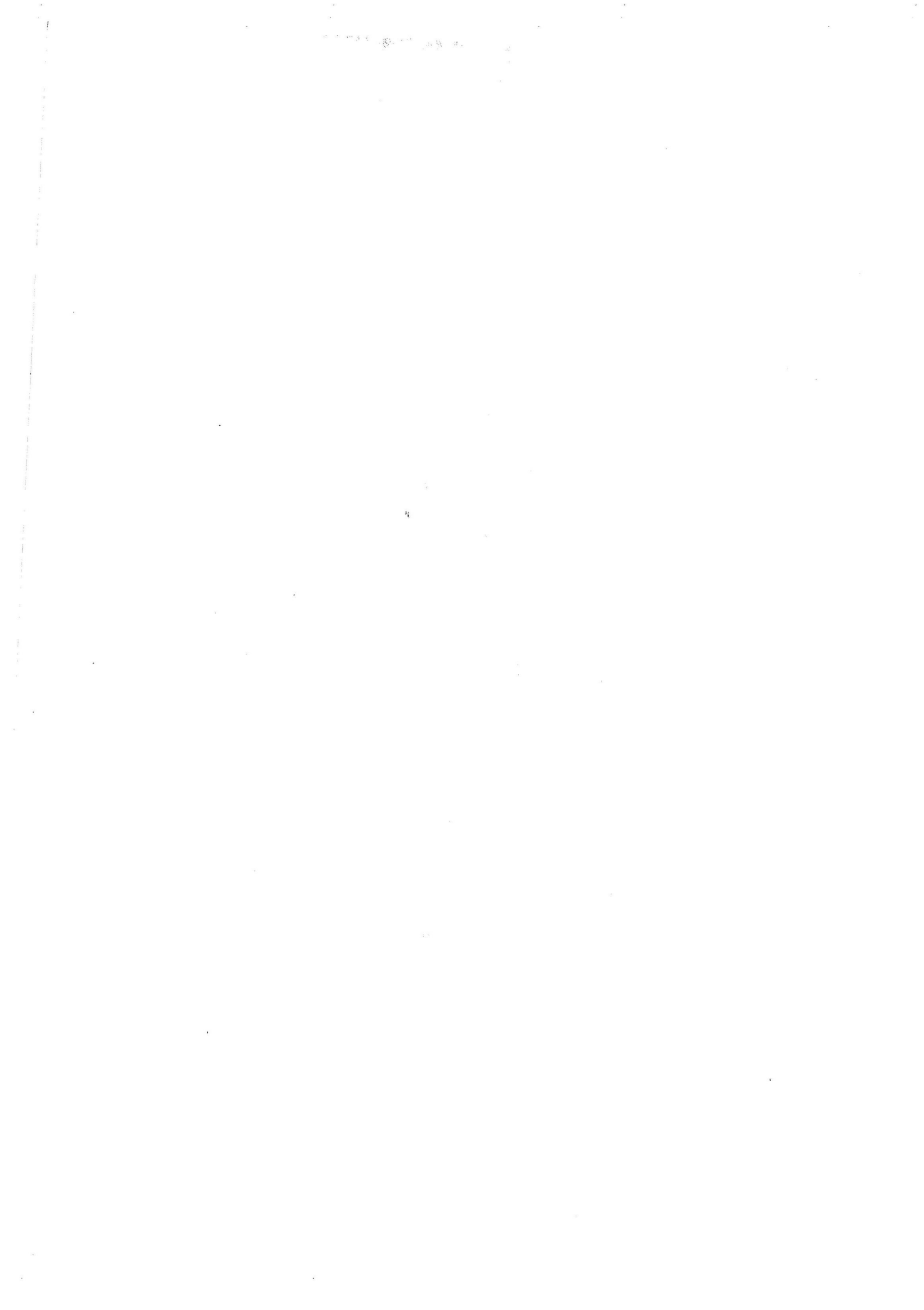
This exam has 8 pages, including the cover page. Please make sure your exam includes each page. Please write your name on *each* page you submit. You will have 50 minutes to complete this exam. **You may not use a calculator**, or consult your textbook, class notes, or any other materials. If you need scratch paper or more space for your answers, please use the back of the pages.

If there is any work on the backs of the pages which you would like to have graded, please indicate this clearly on the front of the page for the corresponding problem.

Show your work for these problems, don't just give an answer. If a question asks you to prove something, please write a complete proof. Unless otherwise stated, you may use any results proved in class or in the textbook, but please make it clear when you are doing so. Unless otherwise stated, you will *not* receive full credit for giving the correct answer with no explanation. You may still earn partial credit even if your final answer is incorrect.

Remember that you are bound by a conduct code, and that you may not look at anyone's paper or let anyone look at your paper.

Question	Points	Score
1	10	
2	30	
3	20	
4	25	
5	15	
Total:	100	



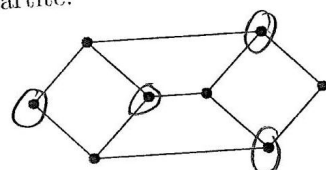
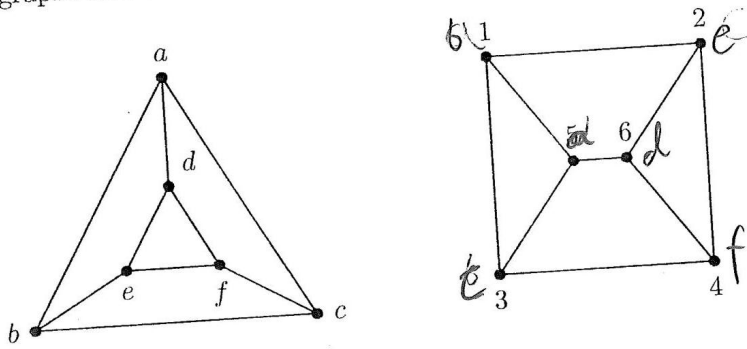


Name: \_\_\_\_\_

27 = 2M

1. [10 pts, 2 points each] Mark each of the following statements as either TRUE or FALSE. For this question you do not need to show any work beyond the final answer.

Be sure to read the questions carefully!

(a) The complete bipartite graph $K_{61,2019}$ has an Euler cycle.	F
(b) There is a graph with 9 vertices in which every vertex has degree 3.	F
(c) The graph $G$ with adjacency matrix $A = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{pmatrix} 2 & 0 & 1 & 0 & 3 \\ 0 & 4 & 0 & 3 & 0 \\ 1 & 0 & 6 & 0 & 5 \\ 0 & 3 & 0 & 2 & 0 \\ 3 & 0 & 5 & 0 & 8 \end{pmatrix} \end{matrix}$ is connected.	F
(d) The graph below is bipartite: 	T
(e) The two graphs below are isomorphic: 	T



Name: \_\_\_\_\_

2. [30 pts] Solve the following recursion relations. In each case, your answer should be a formula for the  $n^{\text{th}}$  term of the sequence in terms of  $n$ .

Show your work. You may use results proved in class or in the textbook, but make it clear how you are getting your answers. A correct answer on its own will not be sufficient for full credit.

(a) [6 pts] Find a general solution to the recursion relation  $a_n = 4a_{n-1} + 5a_{n-2}$ . (That is, find a formula for  $a_n$  in terms of  $n$  and some unknown constants, that will work for any choice of initial conditions.)

$$a_n = 4a_{n-1} + 5a_{n-2}$$

$$\frac{r^n = 4r^{n-1} + 5r^{n-2}}{r^{n-2}} \Rightarrow r^2 = 4r + 5$$

$$r^2 - 4r - 5 = 0$$

$$(r-5)(r+1)$$

$$r = 5, -1$$

$$a_n = 6 \cdot (5)$$

$$a_n = 6 \cdot (5)^n + d \cdot (-1)^n$$

(b) [4 pts] Find the solution to the recurrence relation  $a_n = 4a_{n-1} + 5a_{n-2}$  with initial conditions  $a_0 = 5$  and  $a_1 = 7$ .

$$a_0 = 6(5)^0 + d(-1)^0 = 5 \Rightarrow 6 + d = 5$$

$$d = 5 - 6$$

$$a_1 = 5 \cdot 6 - d = 7$$

$$5 \cdot 6 - 5 + 6 = 7$$

$$6 \cdot 6 = 12$$

$$6 = 2$$

$$d = 3$$

$$a_n = 2(5)^n + 3(-1)^n$$



Name: \_\_\_\_\_

(c) [10 pts] Find the solution to the recurrence relation  $b_n = b_{n-1}^4 b_{n-2}^5$  with initial conditions  $b_0 = 1$  and  $b_1 = 16$ .

$$b_n = e^{a_n} \quad e^{a_n} = (e^{a_{n-1}})^4 \cdot (e^{a_{n-2}})^5$$

$c_0 = 0$   
 $c_1 = \ln(16)$

$$e^{a_n} = e^{4a_{n-1} + 5a_{n-2}}$$

$$a_n = 4a_{n-1} + 5a_{n-2}$$

$$r^2 - 4r - 5 = 0$$

$$a_n = b \cdot (5)^n + d(-1)^n$$

$$b_n = e^{b(s)^n + d(-1)^n}$$

$$b+d=0 \quad b=-d$$

$$b_0 = e^{(b+d)} = 1$$

$$b_1 = e^{5b-d} = 16$$

$$5b-d = \ln(16) \quad d = -\frac{1}{6} \ln(16)$$

$$6b = \ln(16) \quad b = \frac{1}{6} \ln(16)$$

$$b_n = e^{\frac{1}{6} \ln(16) (5^n + (-1)^{n+1})}$$

(d) [10 pts] Find the solution to the recurrence relation  $c_n = 4c_{n-1} + 5c_{n-2} + 16$  with initial conditions  $c_0 = 0$  and  $c_1 = 2$ .

homogeneous sol:

$$c_n = 4c_{n-1} + 5c_{n-2}$$

$$r^2 - 4r - 5 = 0$$

$$r = 5, -1$$

$$c_n = a \cdot (5)^n + b \cdot (-1)^n$$

let  $V_n = c'$

$$c' = 4c' + 5c' + 16$$

$$-8c' = 16$$

$$c' = -2$$

$$c_n = a(5)^n + b(-1)^n - 2$$

$$c_0 = a + b - 2 = 0 \quad b = 2 - a$$

$$c_1 = 5a - b - 2 = 2$$

$$5a - 2 + a - 2 = 2$$

$$6a = 6$$

$$a = 1$$

$$b = 1$$

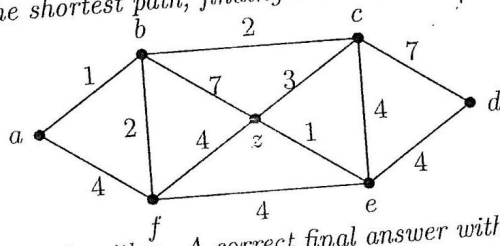
$$c_n = 5^n + (-1)^n - 2$$



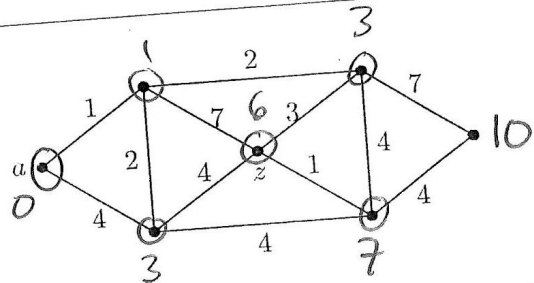
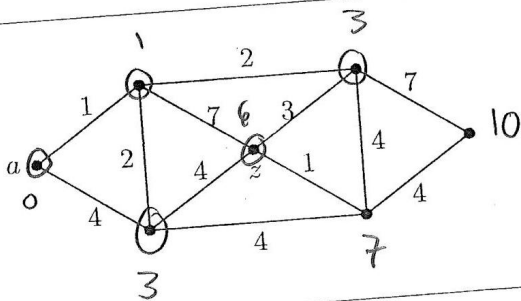
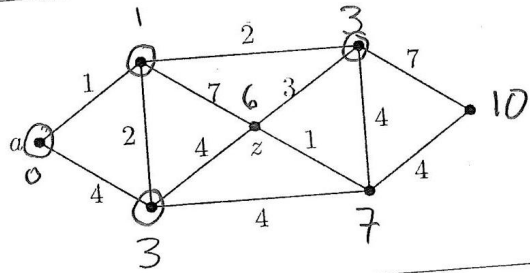
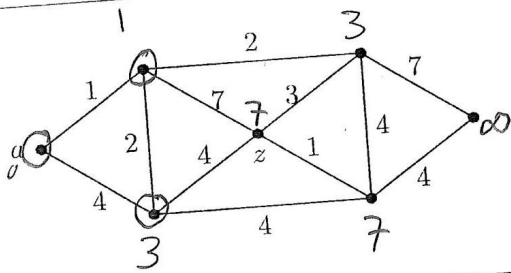
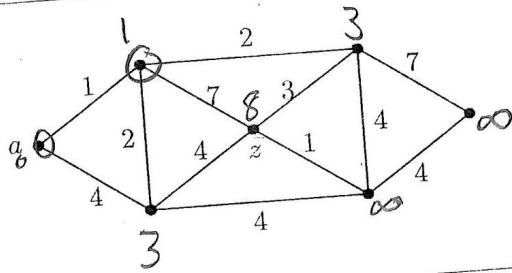
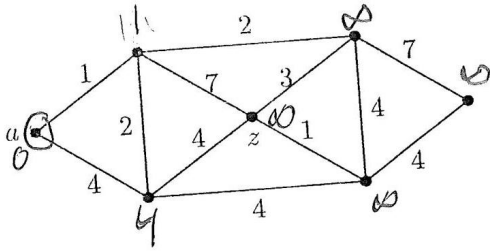


Name: \_\_\_\_\_

3. [20 pts] Use Dijkstra's algorithm to find the length of the shortest path (i.e. the path for which the sum of the labels is as small as possible) between  $a$  and  $z$  in the weighted graph below. You do not need to find the shortest path, finding its length will be sufficient.



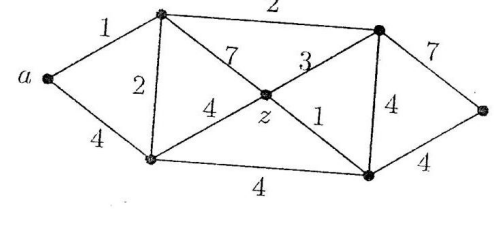
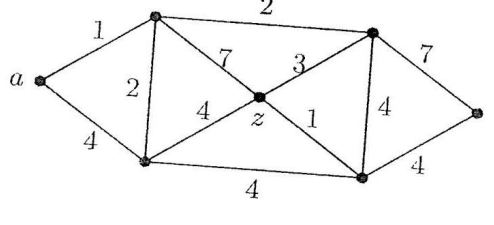
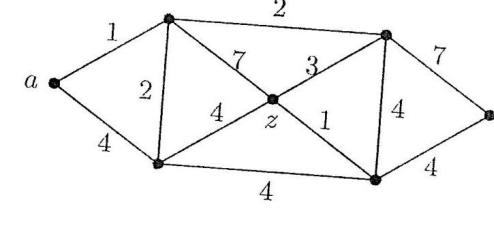
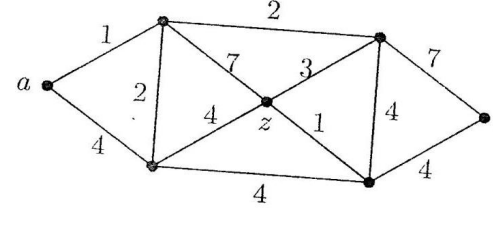
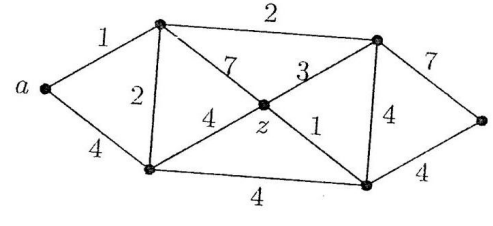
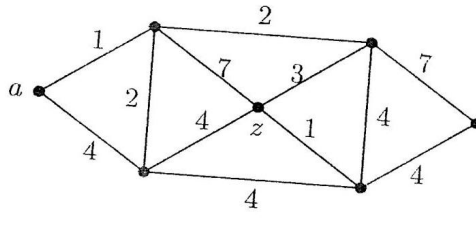
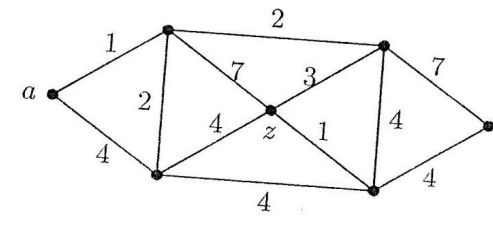
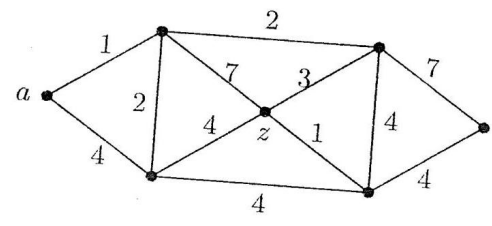
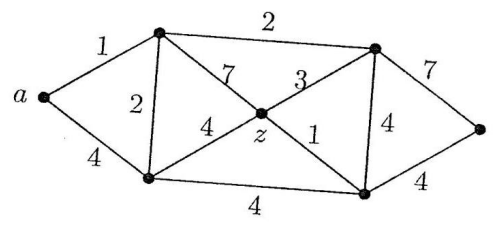
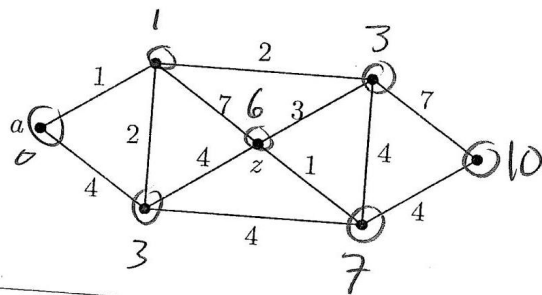
Show each step of Dijkstra's algorithm. A correct final answer with no work shown will not be sufficient for full credit. Use the blank graphs below for your answer. If you make a mistake, clearly cross it out and continue using the next blank graph. There are additional blank graphs on the back of this page.



Answer: 10

Check this box if you used any graphs from the back of the page:





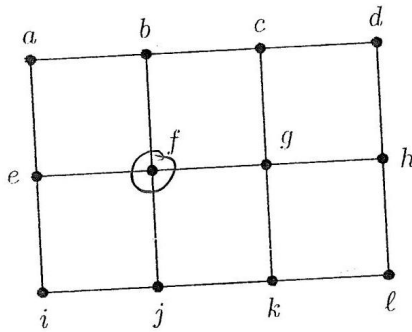
Name: \_\_\_\_\_

4. [25 pts] In each of the following graphs, either find a Hamiltonian cycle (i.e. a cycle which uses every vertex exactly once), or prove that the graph does not have a Hamiltonian cycle.

If the graph does have a Hamiltonian cycle, **CLEARLY** drawing this cycle on the provided graph (so that there's no ambiguity as to which edges are used, and in which order), or listing out the vertices in the order traveled (i.e. writing something like  $(a, b, e, d, c, a)$ ) will be sufficient for full credit.

If the graph does not have a Hamiltonian cycle, you must give an explanation as to why. *Simply drawing diagrams with no explanation will NOT be sufficient for full credit.*

(a) [10 pts]



Number of vertices: 12

Number of edges in Hamiltonian cycle: 12

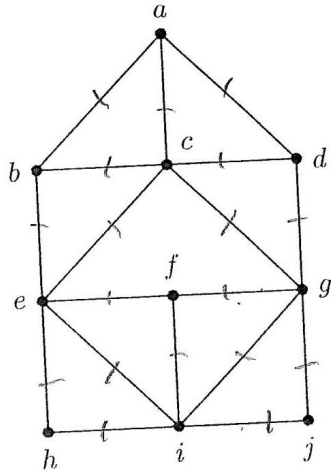
total edges: 13

Looking at the vertex  $f$ , since it has degree 4, 2 of its edges will not be used in the HC, however, the graph will only have 11 edges left to use in the cycle, which is not enough so this graph does not have a Hamiltonian cycle.



Name: \_\_\_\_\_

(b) [15 pts]



# of vertices: 10

# of edges in (HC): 10

total edges: 18

Vertex i has degree 5, meaning 3 will not be used in HC, leaving 15 edges

Vertex c has degree 5, meaning 3 edges will not be used, leaving 12 edges

Vertices e & g each have 3 edges independent from i & c, meaning 1 from each must be used, leaving 10 edges

10 edges remaining, but f will not be connected to,

So there is no Hamiltonian cycle



Name: \_\_\_\_\_

5. [15 pts]

- (a) [10 pts] Let  $G$  be a simple graph with 10 vertices, in which every vertex has degree at least 5. Prove that  $G$  is connected. [Hint: You may want to use the pigeonhole principle.]

$G$  is connected because if there are 10 vertices, then each vertex connects to at least 5 other vertices, then if there is a component, it must contain 6 vertices in it for each vertex to have degree 5, however, this does not leave enough vertices to form another component where each vertex has degree 5 (4 vertices, each vertex will have at most degree 3)  
∴  $G$  is connected

- (b) [5 pts] Draw a simple graph with 10 vertices which is not connected, in which every vertex has degree 4.

