Math 61 Midterm 2

Alvin Phu Nguyen

TOTAL POINTS

67.5 / 100

QUESTION 1

1 8 / 10

- ✓ + **2 pts** a) False
- ✓ + **2 pts** b) False
- ✓ + **2 pts** c) False
- ✓ + **2 pts** d) True
- + **2 pts** e) True

QUESTION 2

30 pts

2.1 a 6 / 6

- ✓ - **0 pts** Correct

2.2 b 4 / 4

- ✓ - **0 pts** Correct (based on answer to a)

2.3 c 2.5 / 10

- ✓ - **7.5 pts** Took log(b_n), no further progress

2.4 d 10 / 10

- ✓ + **10 pts** Correct
- + **2 pts** correct constant on (-1)^n
- + **2 pts** correct constant on 5^n
- + **6 pts** g(n)=-2
- + **2 pts** g(n)=c
- - **2 pts** Minor computational error
- + **0 pts** No significant progress

QUESTION 3

3 20 / 20

- ✓ - **0 pts** Correct

QUESTION 4

25 pts

4.1 a 10 / 10

- ✓ + **10 pts** Correct
- + **0 pts** Did not draw a Hamiltonian cycle

4.2 b 0 / 15

- + **15 pts** Correct
- ✓ + **0 pts** Solution lacked any significant content/tried to use counting method to conclude that no path exists (and believed it worked/used it improperly)
- - **8 pts** Solution is vaguely worded, incomplete or includes false claims that doesn't affect final argument.
- - **3 pts** Does not explain why certain edges must be used/excluded in a potential Hamiltonian cycle.

QUESTION 5

15 pts

5.1 a 2 / 10

- ✓ - **8 pts** Bin 4: The argument does not show any promise of leading to a proof or has not made enough progress to demonstrate the student could have worked out the remaining details. (Usually this is a poor attempt at pigeonhole principle, e.g. one vertex is connected to 5, the next is connected to 5, so that the total is 10; or, concluded # edges > # vertices but tried to say that implies connected)

5.2 b 5 / 5

- ✓ - **0 pts** Correct

MIDTERM 2 (MATH 61)

FRIDAY, MAY 17TH

Name: Alvin Nguyen

ID: 705124129

Circle your discussion section:

Tuesday	Thursday	
2A	[✓ 2B]	TA: Harris Khan
2C	2D	TA: Fred Vu
2E	2F	TA: Matthew Stone

This exam has 8 pages, including the cover page. Please make sure your exam includes each page. Please write your name on *each* page you submit. You will have 50 minutes to complete this exam. **You may not use a calculator**, or consult your textbook, class notes, or any other materials. If you need scratch paper or more space for your answers, please use the back of the pages.

If there is any work on the backs of the pages which you would like to have graded, please indicate this clearly on the front of the page for the corresponding problem.

Show your work for these problems, don't just give an answer. If a question asks you to prove something, please write a complete proof. Unless otherwise stated, you may use any results proved in class or in the textbook, but please make it clear when you are doing so. Unless otherwise stated, you will *not* receive full credit for giving the correct answer with no explanation. You may still earn partial credit even if your final answer is incorrect.

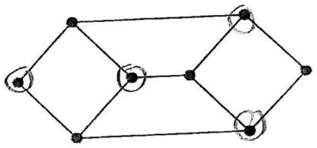
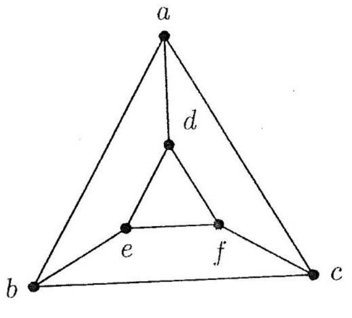
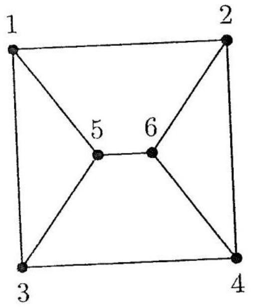
Remember that you are bound by a conduct code, and that you may not look at anyone's paper or let anyone look at your paper.

Question	Points	Score
1	10	
2	30	
3	20	
4	25	
5	15	
Total:	100	

Name:____________________________

1. [10 pts, 2 points each] Mark each of the following statements as either TRUE or FALSE. For this question you do not need to show any work beyond the final answer.

Be sure to read the questions carefully!

(a) The complete bipartite graph $K_{61,2019}$ has an Euler cycle.	F
(b) There is a graph with 9 vertices in which every vertex has degree 3.	F
(c) The graph G with adjacency matrix $A = \begin{pmatrix} 2 & 0 & 1 & 0 & 3 \\ 0 & 4 & 0 & 3 & 0 \\ 1 & 0 & 6 & 0 & 5 \\ 0 & 3 & 0 & 2 & 0 \\ 3 & 0 & 5 & 0 & 8 \end{pmatrix}$ is connected.	F
(d) The graph below is bipartite: 	T
(e) The two graphs below are isomorphic:  	F

Name:______________________________

2. [30 pts] Solve the following recursion relations. In each case, your answer should be a formula for the n^{th} term of the sequence in terms of n.

Show your work. You may use results proved in class or in the textbook, but make it clear how you are getting your answers. A correct answer on its own will not be sufficient for full credit.

(a) [6 pts] Find a *general* solution to the recursion relation $a_n = 4a_{n-1} + 5a_{n-2}$. (That is, find a formula for a_n in terms of n and some unknown constants, that will work for any choice of initial conditions.)

$a_n - 4a_{n-1} - 5a_{n-2} = 0$

$t^2 - 4t - 5 = (t-5)(t+1) = 0 \Rightarrow t = 5, -1$

$$\boxed{a_n = C_1 5^n + C_2(-1)^n}$$

(b) [4 pts] Find the solution to the recurrence relation $a_n = 4a_{n-1} + 5a_{n-2}$ with initial conditions $a_0 = 5$ and $a_1 = 7$.

$5 = C_1 + C_2$

$7 = 5C_1 - C_2$

$\Rightarrow 12 = 6C_1 \Rightarrow C_1 = 2$

$C_2 = 3$

$$\boxed{a_n = 2 \cdot 5^n + 3 \cdot (-1)^n}$$

Name:____________________________________

(c) [10 pts] Find the solution to the recurrence relation $b_n = b_{n-1}^4 b_{n-2}^5$ with initial conditions $b_0 = 1$ and $b_1 = 16$.

Letting $a_n = \log b_n$ $\log b_n = \log(b_{n-1}^4)\log(b_{n-2}^5)$

~~$\log b_n = 20\log(b_{n-1} + b_{n-2})$~~

$b_n = (b_{n-1} + b_{n-2})^{20}$

(d) [10 pts] Find the solution to the recurrence relation $c_n = 4c_{n-1} + 5c_{n-2} + 16$ with initial conditions $c_0 = 0$ and $c_1 = 2$.

$c_n - 4c_{n-1} - 5c_{n-2} = 0$ from (a), $h_n = C_1 5^n + C_2(-1)^n$

Let $y = A \Rightarrow p_n = -2$

$-8A = 16$

$A = -2$

$\Rightarrow c_n = h_n + p_n = C_1 5^n + C_2(-1)^n - 2$

$2 = C_1 + C_2$

$4 = 5C_1 - C_2$

$6 = 6C_1$

$C_1 = 1$ $C_2 = 1$

$\boxed{c_n = 5^n + (-1)^n - 2}$

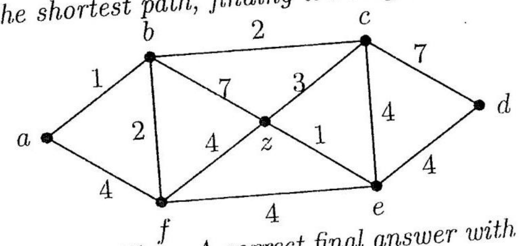
3

$25 + 1 - 2$

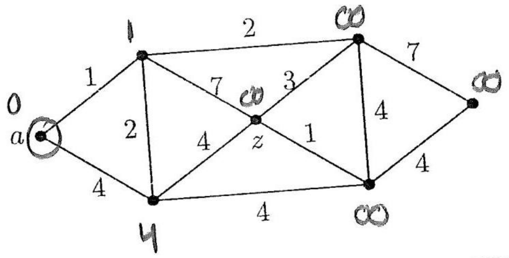
$= 24$

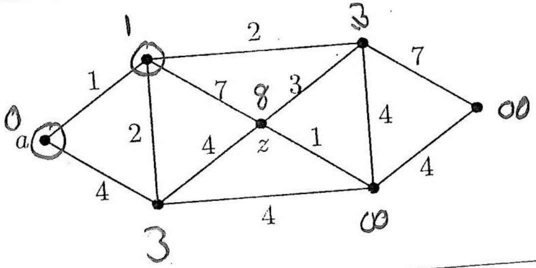
Name:______________________________

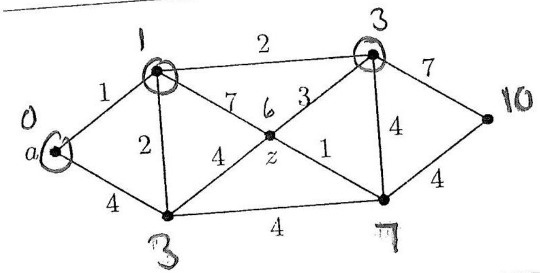
3. [20 pts] Use Dijkstra's algorithm to find the length of the shortest path (i.e. the path for which the sum of the labels is as small as possible) between a and z in the weighted graph below. *You do not need to find the shortest path, finding it's length will be sufficient.*

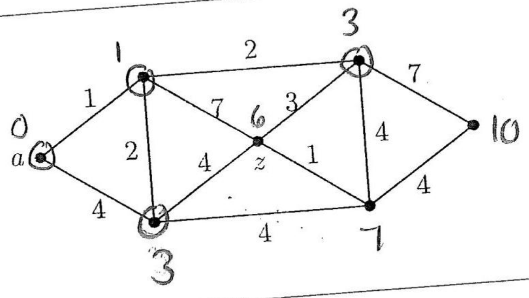


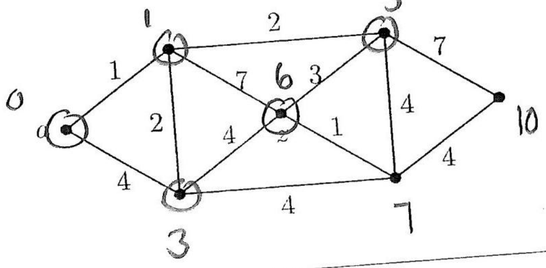
Show each step of Dijkstra's algorithm. A correct final answer with no work shown will not be sufficient for full credit. Use the blank graphs below for your answer. If you make a mistake, clearly cross it out and continue using the next blank graph. There are additional blank graphs on the back of this page.

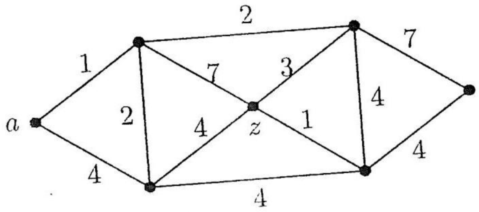






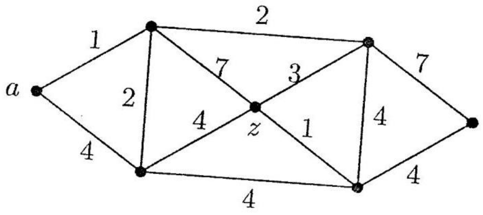






Answer: 6

Check this box if you used any graphs from the back of the page: ☐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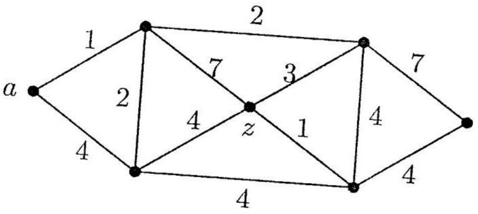
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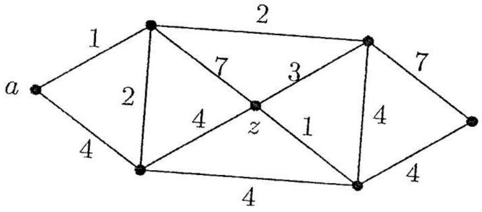
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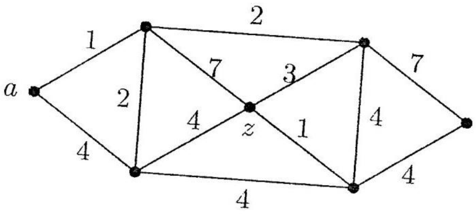
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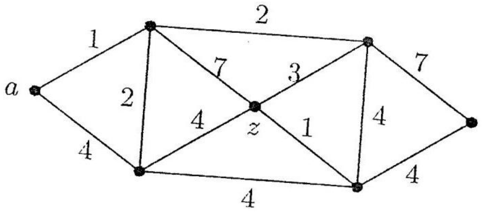
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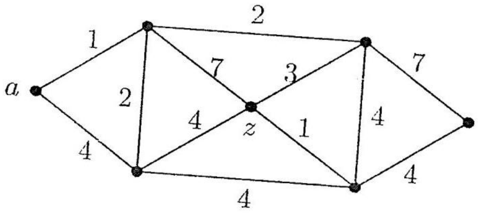
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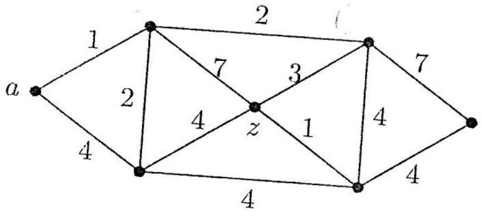
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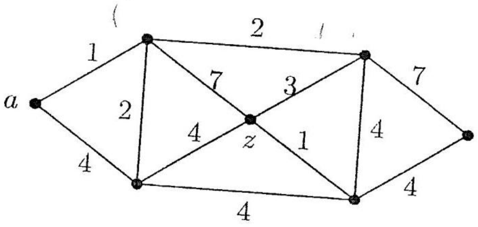
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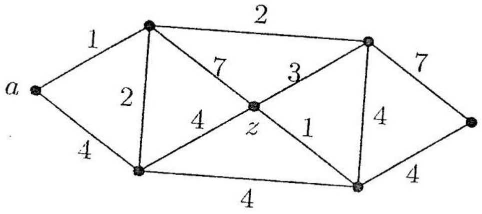
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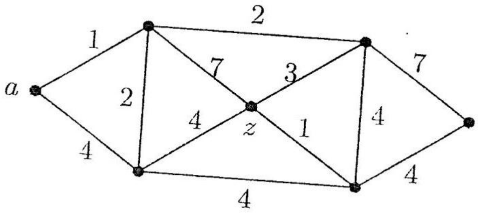
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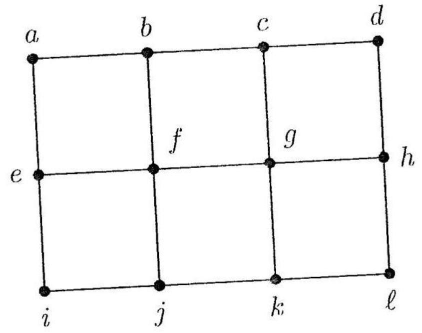
Name:________________________

4. [25 pts] In each of the following graphs, either find a Hamiltonian cycle (i.e. a cycle which uses every vertex exactly once), or prove that the graph does not have a Hamiltonian cycle.

If the graph does have a Hamiltonian cycle, **CLEARLY** drawing this cycle on the provided graph (so that there's no ambiguity as to which edges are used, and in which order), or listing out the vertices in the order traveled (i.e. writing something like (a, b, e, d, c, a)) will be sufficient for full credit.

If the graph does not have a Hamiltonian cycle, you must give an explanation as to why. *Simply drawing diagrams with no explanation will NOT be sufficient for full credit.*

(a) [10 pts]

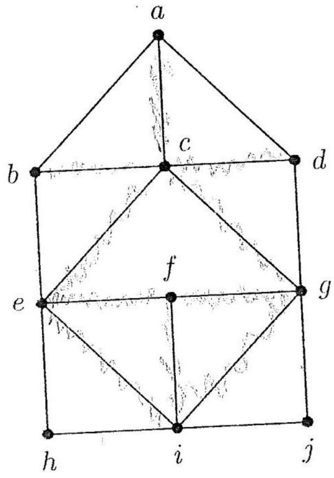


The graph has a Hamiltonian cycle

Path: (a, b, f, g, c, d, h, l, k, j, i, e, a)

Name:____________________________________

(b) [15 pts]



10 verticies
18 edges

There is no Hamiltonian cycle. There are 10 verticies and 18 edges, meaning there are 8 edges we don't use. If there were a Hamiltonian cycle, each vertex would have a degree of 2, so for each vertex that has a degree larger than 2, we exclude $\delta(v) - 2$ edges. If we attempt to do this, we exclude 10 edges, which is a contradiction, because we should only be able to cross out 8 since there are 8 edges we don't use. Therefore, by contradiction, this graph doesn't have a Hamiltonian cycle

from an cycle.

25 edges

10

Name:__________________________________

20

5. [15 pts]

(a) [10 pts] Let G be a *simple* graph with 10 vertices, in which every vertex has degree at least 5. Prove that G is connected. [Hint: You may want to use the pigeonhole principle.]

Since the sum of the degrees is at least 50, there are at least 25 edges in this graph. If we assign each vertex to 5 edges, by the pigeonhole principle at least one edge be associated to a combination of two distinct verticies. This means there will be a path from vertices v, w for and $v, w \in V$ in $G = (V, E)$. Since this is the definition of being connected, G is connected.

Since each vertex has a degree of 5...

(b) [5 pts] Draw a *simple* graph with 10 vertices which is *not* connected, in which every vertex has degree 4.