

Math 61 Midterm 1

TOTAL POINTS

86 / 100

QUESTION 1

1 4 / 10

+ 2 pts a: True

✓ + 2 pts b: False

+ 2 pts c: True

✓ + 2 pts d: True

+ 2 pts e: False

QUESTION 2

35 pts

2.1 a 10 / 10

✓ - 0 pts Correct 3!6!

2.2 b 9 / 10

✓ - 1 pts Wrong exponent on 2

2.3 c 15 / 15

✓ - 0 pts Correct

QUESTION 3

3 15 / 15

✓ + 5 pts Showed reflexivity

✓ + 5 pts Showed symmetry

✓ + 5 pts Showed transitivity

+ 0 pts No credit due

QUESTION 4

4 20 / 20

✓ - 0 pts Correct

QUESTION 5

20 pts

5.1 a 3 / 10

+ 0 pts No serious progress / didn't understand the relevant definitions

✓ + 3 pts Tried something somewhat sensible, but did not make any serious progress towards a formal proof.

+ 6 pts Started with $f(x) = f(y)$, applied g to both sides, concluded $h(x) = h(y)$, but could not properly conclude the result (or struggled to articulate it formally).

+ 10 pts Correct. Started with $f(x) = f(y)$, applied g to both sides, concluded $h(x) = h(y)$, and used injectivity of h to conclude $x = y$. (Or, ran an equivalent proof by contradiction).

5.2 b 10 / 10

+ 0 pts No serious progress

+ 2 pts Bin 4: finite X, Y, Z , one of them is not a function, or g is injective or h is not injective or the composition is not done correctly. Or definitions missing

✓ + 10 pts Bin 5: finite X, Y, Z with f, g, h written out as sets or picture, g clearly not injective and h clearly injective

+ 2 pts Bin 6: some complicated f, g not satisfying the basic conditions necessary for the counterexample (or not at all justified to have these properties)

+ 5 pts Bin 1: g is simple, f is complicated. things like x^2 for g , complicated function for f , X, Y, Z not specified, and h not proven to be injective and nontrivial to see that it is

+ 8 pts Bin 2: both f, g are simple. x^2 , $x^{1/2}$, and worked out why h is injective (or it is clear) and why g is not. Left out what X, Y, Z should be (or incorrectly specified them)

+ 10 pts Bin 3: Fully correct example (X, Y, Z specified or drawn, g not injective and h injective)

MIDTERM 1 (MATH 61)

MONDAY, APRIL 22ND

Name: _____

ID: _____

Circle your discussion section:

Tuesday	Thursday	
2A	2B	TA: Harris Khan
<u>2C</u>	2D	TA: Fred Vu
2E	2F	TA: Matthew Stone

$$\frac{(1-r^{n+1})}{1-r}$$

$$\frac{n \ln(1-r)}{1-r}$$

$n^2 \ln(1-r)$

This exam has 7 pages, including the cover page. Please make sure your exam includes each page. Please write your name on *each* page you submit. You will have 50 minutes to complete this exam. **You may not use a calculator**, or consult your textbook, class notes, or any other materials. If you need scratch paper or more space for your answers, please use the back of the pages.

If there is any work on the backs of the pages which you would like to have graded, please indicate this clearly on the front of the page for the corresponding problem.

Show your work for these problems, don't just give an answer. If a question asks you to prove something, please write a complete proof. Unless otherwise stated, you may use any results proved in class or in the textbook, but please make it clear when you are doing so. Unless otherwise stated, you will *not* receive full credit for giving the correct answer with no explanation. You may still earn partial credit even if your final answer is incorrect.

Remember that you are bound by a conduct code, and that you may not look at anyone's paper or let anyone look at your paper.

Question	Points	Score
1	10	
2	35	
3	15	
4	20	
5	20	
Total:	100	



Name: Marie Chu

1. [10 pts, 2 points each] Mark each of the following statements as either TRUE or FALSE. For this question you do not need to show any work beyond the final answer.

Be sure to read the questions carefully!

(a) The set $\{\{1\}, 3, \{2, 3\}, 2, 3\}$ has cardinality 4.	FALSE
(b) Every relation is either symmetric or antisymmetric. <i>both</i>	FALSE
(c) If R is a relation satisfying $R = (R^{-1})$ ^{bijection} then R is symmetric.	FALSE
(d) The set $f = \{(C, \clubsuit), (B, \heartsuit), (E, \heartsuit), (D, \spadesuit), (A, \clubsuit)\}$ is a <u>function</u> from $X = \{A, B, C, D, E\}$ to $Y = \{\diamond, \clubsuit, \heartsuit, \spadesuit\}$.	TRUE
(e) The set $X = \{1, 2, 3, 4, 5, 6\}$ has more subsets of cardinality 4 than subsets of cardinality 3.	TRUE

$$\frac{C(6,4)}{C(6,3)} = \frac{15}{4}$$



Name: Marta Chu

2. [35 pts] Compute the following quantities. You may leave your answers in terms of exponents and factorials, but do not leave your final answers in terms of $P(n, r)$ or $C(n, r)$ (so $4^{12} \frac{15!}{3!6!2!}$ would be an acceptable final answer, but $P(10, 3)C(18, 7)$ would not).

Show your work. It should be clear how you got your answers.

- (a) [10 pts] The number of permutations of the letters COMPUTER that contain the letters CPU together in any order (so for instance, MTUPCREO would be one such arrangement, but PMTCOREU would not).

First Assume CPU: 3!, then 6! to arrange (CPU) U M T E R

Answer: $3! \cdot 6!$

- (b) [10 pts] The coefficient of $x^6 y^8$ in the expansion of $(3x + 2y^2)^{10}$. [Don't forget that y is squared in this expression.]

Let $a = 3x$ $b = 2y^2$ $c = 4$ using the binomial theorem.

$$(a+b)^n = \sum_{k=0}^n C(n, k) a^k b^{n-k}$$

so the coefficients: $C(10, 6) (3^6) (2^8)$

$$= \frac{10!}{6!4!} (3^6) (2^8)$$



Name: NAME (hw)

- (c) [15 pts] A standard 6-sided die (with sides numbered 1-6) is rolled 10 times in a row. How many possible outcomes are there in which exactly four 5's were rolled, and no two 5's were ever rolled in a row.

[So for example 4562251565 would be one such outcome, but 2544551533 would not. Also order matters, so 4562251565 would be considered a different outcome from 5254152566.]

5 choices
5 choices

5^6 - choose the other dice rolls first, then of the six spots that are five choices (1, 2, 3, 4, 6)

(1) $C(7, 4)$ out of the 7 spaces (to make sure no two fives appear next to each other) pick 4 spots to place the "5's"

$$5^6 (C(7, 4))$$

$$5^6 \frac{7!}{4!(7-4)!} = 5^6 \left(\frac{7!}{4!3!} \right) = \frac{5^6 7!}{4!3!}$$



Name: Marie Chu

3. [15 pts] Let $\mathbb{R}^+ = \{x \in \mathbb{R} \mid x > 0\}$ be the set of positive real numbers. Define a relation R on \mathbb{R}^+ by $(x, y) \in R$ if x/y is a rational number. Prove that R is an equivalence relation.

$$R = \{(x, y) \in \mathbb{R}^+ \times \mathbb{R}^+ \mid (x/y) \in \text{set of rational \#s}\}$$

To be an equivalence relation, R must be reflexive, symmetric, and anti-symmetric.

R is symmetric because for all $x \in \mathbb{R}^+$, $x R x$
because $\frac{x}{x} = 1$ which is rational. so $(x, x) \in R$.

R is symmetric because for all $x, y \in \mathbb{R}^+$, if $(x, y) \in R$
($\frac{x}{y}$ is a rational number) then $(y, x) \in R$ because

$\frac{y}{x}$ would also have to be a rational number since
it is just $(\frac{x}{y})^{-1}$. Thus if $(x, y) \in R$, then $(y, x) \in R$.

R is transitive because for all $x, y, z \in \mathbb{R}^+$, if $(x, y) \in R$
and $(y, z) \in R$ then $(x, z) \in R$ because if $\frac{x}{y}$ is a
rational number and $\frac{y}{z}$ is also a rational number

$\frac{x}{z}$ is also rational since $\frac{x}{z} = \frac{x}{y} \cdot \frac{y}{z}$. The product
of two rational numbers is a rational number.

Thus R is an equivalence relation.



Name: Marie Chu

4. [20 pts] Prove that for any positive integer n :

$$1(1!) + 2(2!) + 3(3!) + \dots + n(n!) = (n+1)! - 1$$

[Hint: Use induction.]

$$\begin{aligned} & ((n+1)+1)! \\ & (n+2)! \end{aligned}$$

We first check the base case $n=1$.

$$1(1!) = (1+1)! - 1$$

$$1 = 2 - 1 = 1 \quad \text{This base case is verified.}$$

We then assume the statement is true for n and
prove it is true for $n+1$.

$$\begin{aligned} & 1(1!) + 2(2!) + 3(3!) + \dots + n(n!) + (n+1)(n+1)! = (n+1)! - 1 + (n+1)(n+1)! \\ & = (n+1)! + (n+1)(n+1)! - 1 \\ & = (n+1)!(1 + n+1) - 1 \\ & = (n+1)!(n+2) - 1 \\ & = (n+2)! - 1 \quad \text{as needed.} \end{aligned}$$

Thus we have verified that the statement is true for $n+1$.

By the principle of mathematical induction the statement is
true for any positive integer n .



Name: Melvin Chu

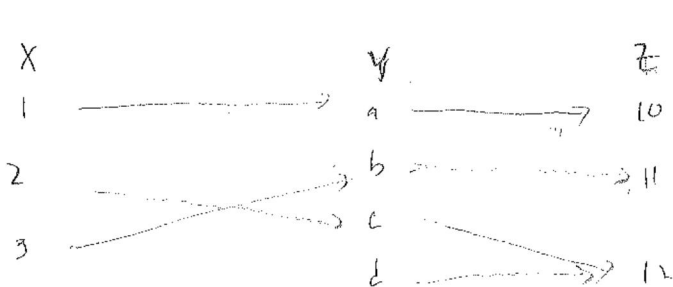
5. [20 pts] Let X, Y and Z be sets, and let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be functions, and let $h = g \circ f$ be the composition of f and g . (That is, h is the function from X to Z defined by $h(x) = g(f(x))$.)

(a) [10 pts] Prove that if h is one-to-one, then f is one-to-one as well. [Hint: Assume that $f(x) = f(y)$, and try to use that to prove that $x = y$.]

If h is one to one, then if $h(x) = h(y)$ $x = y$.
 Thus if $h(x) = g(f(x)) = h(y) = g(f(y))$ so
 $g(f(x)) = g(f(y))$. If $f(x) = f(y)$ then
 $x = y$ since when $h(x) = h(y)$ $x = y$.

(b) [10 pts] Give a counterexample to show that it is possible for h to be one-to-one while g is not one-to-one. That is, give examples of sets X, Y and Z and functions $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ such that $h = g \circ f$ is one-to-one, but g is *not* one-to-one.

$X = \{1, 2, 3\}$ $Y = \{a, b, c, d\}$
 $Z = \{10, 11, 12\}$



$f \{ (1, a), (2, b), (3, c) \}$
 $g \{ (a, 10), (b, 11), (c, 11), (d, 12) \}$
 $h \{ (1, 10), (2, 11), (3, 11) \}$

As it is shown h is one to one but g is not one to one because 2 values map to 11.

