Math 61-1 Midterm 2 version a



TOTAL POINTS

43 / 50

QUESTION 1

Multiple choice 10 pts

1.1 2 / 2

√ - 0 pts Correct (d)

- 2 pts Incorrect

1.2 2/2

√ - 0 pts Correct (a)

- 2 pts Incorrect

1.3 2/2

√ - 0 pts Correct (c)

- 2 pts Incorrect

- 2 pts No Answer

1.4 2/2

√ - 0 pts Correct (b)

- 2 pts Incorrect

1.5 2/2

√ - 0 pts Correct (d)

- 2 pts Incorrect

- 2 pts No Answer

QUESTION 2

Short answer 10 pts

2.1 2 / 2

√ - 0 pts Correct (176)

- 2 pts Incorrect

- 2 pts No answer

2.2 2/2

√ - 0 pts Correct (34650)

- 2 pts Incorrect

2.3 0/2

- 0 pts Correct (21)

√ - 2 pts Incorrect

- 2 pts No answer

2.4 2/2

√ - 0 pts Correct (960)

- 2 pts Incorrect

- 0.5 pts Gave term, not just coefficient

- 1.5 pts Didn't multiply by 8 (120) or similar

- 2 pts No answer

2.5 2/2

√ - 0 pts Correct (302400)

- 2 pts Incorrect

- 2 pts No answer

QUESTION 3

Euler paths 10 pts

3.1 criteria for euler path 5/5

√ + 5 pts Correct

+ 2 pts Euler cycle criterion

+ 1 pts Euler cycle criterion (missing connected, or other mistake)

+ 2 pts Reduction to graph with even degrees

+ 1 pts Reduction to graph with even degrees (with nistake)

+ 1 pts Correct explanation of how to get Euler path from Euler cycle

+ 0 pts Incorrect

3.2 application of euler paths 6/5

√ - 0 pts Correct

√ + 1 pts Click here to replace this description.

+ 2 pts Click here to replace this description.

QUESTION 4

4 Pigeon hole 10 / 10

- √ 0 pts Correct
 - 3 pts Incorrect Partition
 - 3 pts No Pigeonhole
 - 2 pts Minor error
 - 10 pts Blank

QUESTION 5

Hypercube 10 pts

5.1 recurrence for edges 0 / 4

- O pts Correct
- √ 4 pts empty
 - 2 pts large mistake
 - 2 pts you are assuming the desired conclusion
 - 1 pts need to explain how the hypercube it built

out of smaller ones

- 3 pts can't just do examples
- 1 pts incomplete
- 3 pts I don't see how this shows that the

recurrence is true

5.2 number of edges 4/6

- 0 pts Correct
- 1 pts Need to use induction to show that formula is

true

- 4 pts wrong answer, need to use iteration
- 6 pts empty
- 3 pts wrong answer, this is why you need to use

induction to show that answer is true

- 4 pts incomplete
- √ 2 pts wrong answer
 - your 2^k -1 should be a k instead.

Midterm 2

Name:

Student ID:

Section: Tuesday: Thursday:

1A 1B TA: Albert Zheng

1C 1D TA: Benjamin Spitz

1E 1F TA: Eilon Reisin-Tzur

Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. You may not use calculators, books, notes, or any other material to help you. Please make sure your phone is silenced and stowed where you cannot see it. Remember that you are bound by a conduct code, and that you may not look at anyone's paper or let anyone look at your paper, nor may you work on the exam after it has ended. There is some scratch paper at the back of the exam, if you do work there indicate so on the relevant question. Do not write on the backs of the pages. Please circle or box your final answers.

Please get out your id and be ready to show it when you turn in your exam.

Please do not write below this line.

Question	Points	Score
1	10	
2	10	
3	10	""
4	10	
5	10	
Total:	50	

- 1. K_n has an Euler cycle:
 - (a) For every n
 - (b) For no n
 - (c) For n even
 - (d) For n odd

2. If X is a set with 10 elements and Y is a set with 3 elements and $f: X \to Y$:

- (a) there are at least 4 distinct elements of X, x_1, x_2, x_3, x_4 with $f(x_1) = f(x_2) = f(x_3) = f(x_4).$
- (b) it is possible that there are not 4 distinct elements of X, x_1 , x_2 , x_3 , x_4 with $f(x_1) = f(x_2) = f(x_3) = f(x_4)$.
- (c) there are at least 5 distinct elements of X, x_1, x_2, x_3, x_4, x_5 with $f(x_1) = f(x_2) = f(x_3) = f(x_4) = f(x_5).$
- (d) f could be one-to-one

3. If $s_0 = 2$ and $s_1 = 1$ and for $n \ge 2$ $s_n = s_{n-1} + s_{n-2}$ then $s_n = 3$

(a)
$$\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n$$

(b)
$$\frac{1}{\sqrt{5}} \left(\frac{1 + \sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1 - \sqrt{5}}{2} \right)^n$$

$$\left(c \right) \left(\frac{1 + \sqrt{5}}{2} \right)^n + \left(\frac{1 - \sqrt{5}}{2} \right)^n$$

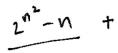
(d) none of the above

Question 1 continued...

- 4. The number of ways of arranging a math book, a computer science book, and an English book on a shelf is: history book as well
 - (a) 4
 - (b) 4!
 - (c) 1+2+3+4
 - (d) 4-3+2-1
- 5. The number of relations on a set with n elements that are both symmetric and reflexive is:
 - (a) 2^{n^2}
 - (b) $2^{n^2-n} + 2^{\frac{n^2+n}{2}} 2^{\frac{n^2-n}{2}}$
 - (c) $2^{n^2} + 2^{\frac{n^2+n}{2}} 2^{\frac{n^2-n}{2}}$
 - d none of the above







- 2. In this question write down your answer, no need for any justification. Leave your answers in a form involving factorials, P(n, m), $\binom{n}{m}$, exponents, etc.
 - (a) (2 points) What is the number of solutions to the equation $x_1 + x_2 + x_3 = 20$ where $x_3 \le 10$ and x_1, x_2, x_3 are nonnegative integers?

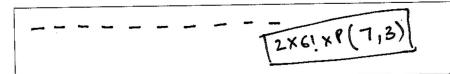
$$\begin{pmatrix} 22\\2 \end{pmatrix} - \begin{pmatrix} 11\\2 \end{pmatrix}$$

(b) (2 points) What is the number of ways of rearranging the letters of the word "MISSISSIPPI"?

(c) (2 points) What is the number of binary strings of length 6 that don't contain 11 as a substring?

(d) (2 points) What is the coefficient of x^7y^3 in the polynomial $(x+2y)^{10}$?

(e) (2 points) I have 10 distinct books. Two are by Ernest Hemingway and three are by Virginia Woolf. How many ways are there to arrange these books on a shelf if the ones by Hemingway must be next to each other and the ones by Woolf must not be next to each other?



3. (a) (5 points) Show that if a connected graph has exactly 2 vertices of odd degree, then there is a path in the graph that visits each edge exactly once. (Feel free to use what we know about Euler cycles from lecture)

If a graph has exactly 2 vertices a ef old degree, then the all the other vertices have even legree.

Construct an edge 'e' between the his vertice of odd degree, say vi and vz. ... vi and vz now have an even largee.

Since, all the vertices now have an even degree in this new graph say G', ... There is an enter cyleacle from vi to vay such that vi... & vz evi.

If those is an Euler cycle from v, , then there must be a path from vi to vz and then from vz to vi through e that visits all the edges.

There is an Euler path from voi to vz.

There is a path from vi to vz in the original graph which visits each edge exactly once.

(b) (5 points) A house has a front door and a back door and no other doors leading outside. Inside each room of the house, there are (exactly) two doors that each go to other rooms of the house ¹ (and the only way to enter or leave a room is through a door). Each room can be reached from the front door. Show that it is possible to go in through the front door and leave through the back door while going through each door in the house exactly once. Feel free to use the previous part of this problem.

The above problem can be modeled as a graph. Ever The house is the graph where every door is a vertos and each norm is an edge. Since, the bout down opens who a norm but has rosm an in other side connect it will be outside the house), there fre it has an odd degree. Similary, for the back dear. E.2 Every other door on the house, is between how different nome and kny has a degree of Zunich is even. 8 And SINCE all the rooms are connected, the graph is connected. * a Since, the funt don't (vertice) and back don (vertice) an odd degree and every other daw (vertex) how an even degree, .. There is an owner path from the first door to the gran door. That is, there is a path from the four door to the back door each nom exactly once.

The doroom just
after and
befre the
front and
back dos
have dogr

¹To clarify the rooms just inside the front and the back doors have three doors, one of them is the front/back door, and the other two are the doorways leading to other rooms of the house

4. (10 points) Suppose that six distinct integers are selected from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Prove that at least two of these six integers sum to 11.

We can partition the above set as follows, \$1,104, \$2,94, \$3,81, \$4,73, \$5,63

There are 5 such partition.

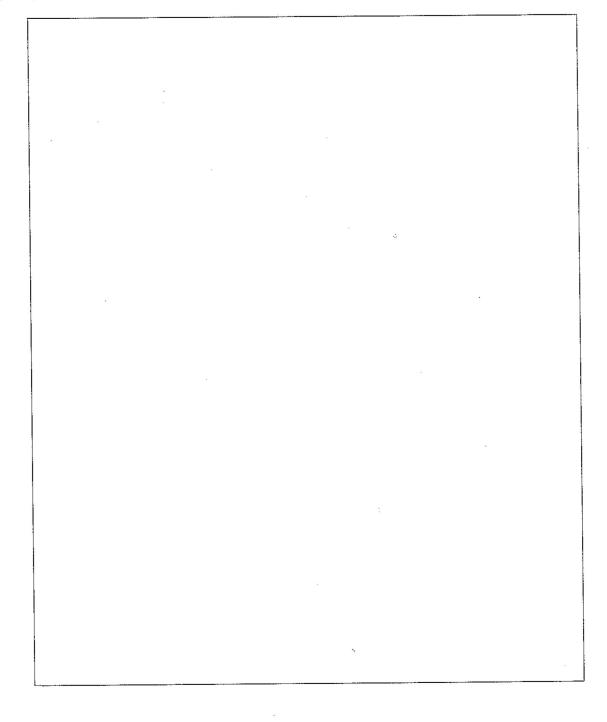
If we have any 6 numbers and assign them to these parition, there will be at least one partition with 2 numbers by the Pigeonhole principle.

The sum of the numbers in each partition is 11.

Since, there at least one parition will have 2 numbers that sum up to 11,

threfor which sum & up to 11.

- 5. Recall the n-dimensional hypercube. This is a graph with vertices labeled by binary strings of length n, with an edge between two vertices if they differ in exactly one digit. Let c_n be the number of edges in the n-dimensional hypercube.
 - (a) (4 points) Show that c_n satisfies the recurrence $c_n = 2^{n-1} + 2c_{n-1}$



(b) (6 points) Solve your recurrence from the previous part of this question to find a formula for c_n .

$$(n = 2^{n-1} + 2 \cdot (n-1)$$

$$(n = 2^{n-1} + 2^{n} + 2 \cdot (n-2)$$

$$\Rightarrow (n = 2^{n-1} + 2^{n} + 2^{$$

· I STER SMEK

Continued as next page

This page has been left intentionally blank. You may use it as scratch paper. It will not be graded unless indicated very clearly here and next to the relevant question.

$$20 - 11 = 9$$

$$20 - 11 = 9$$

$$(20 + 3 - 1)$$

$$(20 + 3 - 1)$$

$$(20 + 3 - 1)$$

$$(20 + 3 - 1)$$

$$(21)$$

$$2 + 2 + 2^{2} + 2^{3}$$