Math 61-1 Midterm 2 version a

DEVIN YERASI

TOTAL POINTS

25 / 50

QUESTION 1

Multiple choice 10 pts

1.1 2 / 2

✓ - **0 pts** **Correct (d)**

- **2 pts** Incorrect

1.2 2 / 2

✓ - **0 pts** **Correct (a)**

- **2 pts** Incorrect

1.3 0 / 2

- **0 pts** Correct (c)

✓ - **2 pts** **Incorrect**

- **2 pts** No Answer

1.4 2 / 2

✓ - **0 pts** **Correct (b)**

- **2 pts** Incorrect

1.5 0 / 2

- **0 pts** Correct (d)

✓ - **2 pts** **Incorrect**

- **2 pts** No Answer

QUESTION 2

Short answer 10 pts

2.1 0 / 2

- **0 pts** Correct (176)

- **2 pts** Incorrect

✓ - **2 pts** **No answer**

2.2 2 / 2

✓ - **0 pts** **Correct (34650)**

- **2 pts** Incorrect

2.3 0 / 2

- **0 pts** Correct (21)

✓ - **2 pts** **Incorrect**

- **2 pts** No answer

2.4 2 / 2

✓ - **0 pts** **Correct (960)**

- **2 pts** Incorrect

- **0.5 pts** Gave term, not just coefficient

- **1.5 pts** Didn't multiply by 8 (120) or similar

- **2 pts** No answer

2.5 0 / 2

- **0 pts** Correct (302400)

✓ - **2 pts** **Incorrect**

- **2 pts** No answer

QUESTION 3

Euler paths 10 pts

3.1 criteria for euler path 4 / 5

+ **5 pts** Correct

✓ + **2 pts** **Euler cycle criterion**

+ **1 pts** Euler cycle criterion (missing connected, or other mistake)

✓ + **2 pts** **Reduction to graph with even degrees**

+ **1 pts** Reduction to graph with even degrees (with mistake)

+ **1 pts** Correct explanation of how to get Euler path from Euler cycle

+ **0 pts** Incorrect

3.2 application of euler paths 6 / 5

✓ - **0 pts** **Correct**

✓ + **1 pts** **Click here to replace this description.**

+ **2 pts** Click here to replace this description.

QUESTION 4

4 Pigeon hole **0 / 10**

- **- 0 pts** Correct
- **- 3 pts** Incorrect Partition
- **- 3 pts** No Pigeonhole
- **- 2 pts** Minor error
- ✓ **- 10 pts Blank**

QUESTION 5

Hypercube 10 pts

5.1 recurrence for edges **0 / 4**

- **- 0 pts** Correct
- ✓ **- 4 pts empty**
- **- 2 pts** large mistake
- **- 2 pts** you are assuming the desired conclusion
- **- 1 pts** need to explain how the hypercube it built out of smaller ones
- **- 3 pts** can't just do examples
- **- 1 pts** incomplete
- **- 3 pts** I don't see how this shows that the recurrence is true

5.2 number of edges **5 / 6**

- **- 0 pts** Correct
- ✓ **- 1 pts Need to use induction to show that formula is true**
- **- 4 pts** wrong answer, need to use iteration
- **- 6 pts** empty
- **- 3 pts** wrong answer, this is why you need to use induction to show that answer is true
- **- 4 pts** incomplete
- **- 2 pts** wrong answer

J. Cameron Math 61 Mon, February 25th, 2019

Midterm 2

Name: Devin Ash ~lerasi

Student ID: 305 167 818

Section:

Tuesday:	Thursday:	
1A	1B	TA: Albert Zheng
1C	1D	TA: Benjamin Spitz
1E	1F	TA: Eilon Reisin-Tzur

Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. **You may not use calculators,** books, notes, or any other material to help you. Please make sure **your phone is silenced** and stowed where you cannot see it. Remember that you are bound by a conduct code, and that you may not look at anyone's paper or let anyone look at your paper, nor may you work on the exam after it has ended. There is some scratch paper at the back of the exam, if you do work there indicate so on the relevant question. Do not write on the backs of the pages. Please circle or box your final answers.

Please get out your id and be ready to show it when you turn in your exam.

Please do not write below this line.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

$\frac{2}{\sqrt{5}} + 1 + \frac{2}{\sqrt{5}} - 1$

1. (10 points) Circle the correct answer (only one answer is correct for each question)

   1. K_n has an Euler cycle:
      (a) For every n
      (b) For no n
      (c) For n even
      (d) For n odd

$\alpha\left(\frac{1+\sqrt{5}}{2}\right)^3 + \beta\left(\frac{1-\sqrt{5}}{2}\right)^3$

$a_2 = 3 = \alpha(2+\sqrt{5}) + \beta(2-\sqrt{5})$

$a_3 = 4 = \alpha$

$\frac{2\sqrt{5}}{\sqrt{5}} = 2$

$\frac{2+\sqrt{5}}{\sqrt{5}} - \frac{2-\sqrt{5}}{\sqrt{5}} =$

1

3, 3, 3

   2. If X is a set with 10 elements and Y is a set with 3 elements and $f : X \to Y$:
      (a) there are at least 4 distinct elements of X, x_1, x_2, x_3, x_4 with $f(x_1) = f(x_2) = f(x_3) = f(x_4)$.
      (b) it is possible that there are *not* 4 distinct elements of X, x_1, x_2, x_3, x_4 with $f(x_1) = f(x_2) = f(x_3) = f(x_4)$.
      (c) there are at least 5 distinct elements of X, x_1, x_2, x_3, x_4, x_5 with $f(x_1) = f(x_2) = f(x_3) = f(x_4) = f(x_5)$.
      (d) f could be one-to-one

$(1-\sqrt{5})(1-\sqrt{5}) = 1 - \sqrt{5} - \sqrt{5} + 5$

$(6-2\sqrt{5})(1-\sqrt{5})$

   3. If $s_0 = 2$ and $s_1 = 1$ and for $n \geq 2$ $s_n = s_{n-1} + s_{n-2}$ then $s_n =$
      (a) $\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n$
      (b) $\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$
      (c) $\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$
      (d) none of the above

$6 - 6\sqrt{5} - 2\sqrt{5} + 10$

$16 - 8\sqrt{5}$

$s_2 = 1 + 2 = 3$

$s_3 = s_2 + s_1 = 3 + 1 = 4$

$\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)\left(\frac{1+\sqrt{5}}{2}\right)\left(\frac{1+\sqrt{5}}{2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)\left(\frac{1-\sqrt{5}}{2}\right)\left(\frac{1-\sqrt{5}}{2}\right)\right]$ 8 8

$(1+\sqrt{5})(1+\sqrt{5}) = 1 + 2\sqrt{5} + 5$

$(6+2\sqrt{5})(1+\sqrt{5}) = 6 + 6\sqrt{5} + 2\sqrt{5} + 2(5) =$

$\frac{16+8\sqrt{5}}{8} - \frac{16-8\sqrt{5}}{8} = \frac{16\sqrt{5}}{8}$

$s_n - s_{n-1} - s_{n-2} = 0$

$r^2 - r - 1 = 0$

$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$

Question 1 continues on the next page...

Question 1 continued...

4 books

4. The number of ways of arranging a math book, a computer science book, and an English book on a shelf is:

   (a) 4
   (b) 4! $4 \times 3 \times 2 \times 1$
   (c) $1+2+3+4$
   (d) $4-3+2-1$

5. The number of relations on a set with n elements that are both symmetric and reflexive is:

   (a) 2^{n^2}
   (b) $2^{n^2-n} + 2^{\frac{n^2+n}{2}} - 2^{\frac{n^2-n}{2}}$ 2
   (c) $2^{n^2} + 2^{\frac{n^2+n}{2}} - 2^{\frac{n^2-n}{2}}$ $2+2-1=3$
   (d) none of the above

2. In this question write down your answer, no need for any justification. Leave your answers in a form involving factorials, $P(n, m)$, $\binom{n}{m}$, exponents, etc.

(a) (2 points) What is the number of solutions to the equation $x_1 + x_2 + x_3 = 20$ where $x_3 \leq 10$ and x_1, x_2, x_3 are nonnegative integers?

Chose 10 for x3 then 10 for x_1 or x_2

(b) (2 points) What is the number of ways of rearranging the letters of the word "MISSISSIPPI"?

M I S P
I S P
I S
I S

$\frac{11!}{4!4!2!}$

(c) (2 points) What is the number of binary strings of length 6 that don't contain 11 as a substring?

_0_0_0 or

$2\times2\times2=8$

(d) (2 points) What is the coefficient of x^7y^3 in the polynomial $(x+2y)^{10}$?

$\sum_{k=0} \binom{n}{k} x^{n-k} y^k$ x^7y^3 $k=3$

$(x+2y)^{10}$

$\binom{10}{3}(1)^7(2)^3 = \binom{10}{3}(2)^3$

(e) (2 points) I have 10 distinct books. Two are by Ernest Hemingway and three are by Virginia Woolf. How many ways are there to arrange these books on a shelf if the ones by Hemingway must be next to each other and the ones by Woolf must not be next to each other?

$(2\times 9!) - 3!(7)(6!) = 2(9!) - 3!\,7!$

_B_B_B_ _ _ _

B_3 _ _ _ _ _

_ _ _ _ _ _ _ _ _ H H

B_3 _ _ _ _ _ _

3. (a) (5 points) Show that if a connected graph has exactly 2 vertices of odd degree, then there is a path in the graph that visits each edge exactly once. (Feel free to use what we know about Euler cycles from lecture)

If a connected graph has two vertices of odd degree, an edge (e_1) can be added between these two vertices (v_1, v_n) such that every vertex has an even degree and there is a Euler cycle, and hence a Euler path (a path that visits each edge exactly once). This new edge forms a new Graph G' of which G is a subgraph.

Since each vertex (v_1, v_2) in G is of an odd degree, there is one more path going out than in for v_1, and one more path going in than out for v_2. One can loop from v_1 to v_1 an arbitrary amount of times $\frac{\delta(v_1)-1}{2}$ and then leave v_1 to go to the next vertex, which if it is not v_n has an even degree and hence can be left to a next node without using the same edge. This follows for all vertices (v_i) $| \ i \neq 1, n$ until it reaches v_n. At this point, there is one path going in and an arbitrary # of cycles at v_n, which can be traversed without repeating an edge, forming a euler path.

Question 3 continues on the next page...

F

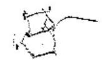
B

(b) (5 points) A house has a front door and a back door and no other doors leading outside. Inside each room of the house, there are (exactly) two doors that each go to other rooms of the house [1] (and the only way to enter or leave a room is through a door). Each room can be reached from the front door. Show that it is possible to go in through the front door and leave through the back door while going through each door in the house exactly once. Feel free to use the previous part of this problem.

This problem can be modeled with a graph, with each ~~room~~ door being a vertex (v) and each room leading to 2 more doors being edges (e). Since the front door (v_1) and back door (v_n) each have 3 paths either leading in to it or from it (respectively), they have odd degrees. However, every other vertex (v_i) | $i \neq 1, n$ leads to a room with 2 more doors indicating $\delta(v_i) = 2$ which is even. As shown by the previous problem, since there are exactly 2 vertexes of an odd degree and an arbitrary # of vertexes with even degrees, there exists a euler path such that one could go in through the front (v_1) and finish at the back (v_2) while visiting each edge (each room) exactly once.

[1] To clarify the rooms just inside the front and the back doors have three doors, one of them is the front/back door, and the other two are the doorways leading to other rooms of the house

4. (10 points) Suppose that six distinct integers are selected from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Prove that at least two of these six integers sum to 11.



12

4+2

3

5. Recall the n-dimensional hypercube. This is a graph with vertices labeled by binary strings of length n, with an edge between two vertices if they differ in exactly one digit. Let c_n be the number of edges in the n-dimensional hypercube.

(a) (4 points) Show that c_n satisfies the recurrence $c_n = 2^{n-1} + 2c_{n-1}$

Base Case: $(n=1)$ $C_0 = 1 = 2^0 + 2(0) = 1$

$n=2$ $C_2 = 2^1 + 2(1) = 4 = 4$

Induction step: assume true for $\leq n$

$C_{n+1} =$

Question 5 continues on the next page...

Question 5 continued...

(b) (6 points). Solve your recurrence from the previous part of this question to find a formula for c_n.

$$C_n = 2^{n-1} + 2C_{n-1}$$

$$C_n = 2^{n-1} + 2(2^{n-2} + 2C_{n-2})$$

$$C_n = 2^{n-1} + 2(2^{n-2}) + 2^2(C_{n-2})$$

$$C_n = 2^{n-1} + 2(2^{n-2}) + 2^2(2^{n-3} + 2C_{n-3})$$

$$C_n = 2^{n-1} + 2^{n-1} + \cancel{2^2}(2^{n-1}) + 2^3(C_{n-3})$$

$$C_n = n(2^{n-1}) + \cancel{2^n(C_0)}$$

$$\boxed{C_n = n(2^{n-1})}$$

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$$\left(\frac{1+\sqrt{5}}{2}\right)\left(\frac{1+\sqrt{5}}{2}\right)\left(\frac{1+\sqrt{5}}{2}\right)$$

$$\left(\frac{6+2\sqrt{5}}{4}\right)\left(\frac{1+\sqrt{5}}{2}\right)=\left(\frac{16+8\sqrt{5}}{8}\right)=$$

$$2+\sqrt{5}$$

$$\left(\frac{1-\sqrt{5}}{2}\right)\left(\frac{1-\sqrt{5}}{2}\right)\left(\frac{1-\sqrt{5}}{2}\right)=\left(\frac{6-2\sqrt{5}}{4}\right)\left(\frac{1-\sqrt{5}}{2}\right)=\left(\frac{16-8\sqrt{5}}{8}\right)$$

$$S_n = S_{n-1}+S_{n-2}$$

$$\frac{2+\sqrt{5}}{\sqrt{5}} \quad \cancel{+} \quad \frac{2-\sqrt{5}}{\sqrt{5}} \qquad \frac{2\sqrt{5}}{\sqrt{5}} \quad 2$$