Math 61-1 Midterm 2 version b

CATHERINE XU

TOTAL POINTS

45 / 50

QUESTION 1

Multiple choice 10 pts

1.1 2 / 2

✓ **- 0 pts Correct (d)**

- 2 pts Incorrect

1.2 2 / 2

✓ **- 0 pts Correct (d)**

- 2 pts Incorrect

- 2 pts No Answer

1.3 2 / 2

✓ **- 0 pts Correct (b)**

- 2 pts Incorrect

1.4 2 / 2

✓ **- 0 pts Correct (c)**

- 2 pts Incorrect

1.5 2 / 2

✓ **- 0 pts Correct (b)**

- 2 pts Incorrect

QUESTION 2

Short answer 10 pts

2.1 2 / 2

✓ **- 0 pts Correct (960)**

- 2 pts Incorrect

- 0.5 pts Gave term, not just coefficient

- 1.5 pts Didn't multiply by 8 (120) or similar

2.2 2 / 2

✓ **- 0 pts Correct (34650)**

- 2 pts Incorrect

2.3 1 / 2

- 0 pts Correct (302400)

- 2 pts Incorrect

- 2 pts No answer

- 1 Point adjustment

Close

2.4 0 / 2

- 0 pts Correct (21)

✓ **- 2 pts Incorrect**

- 2 pts No answer

2.5 0 / 2

- 0 pts Correct (176)

✓ **- 2 pts Incorrect**

- 2 pts No answer

QUESTION 3

Euler paths 10 pts

3.1 criteria for euler paths 5 / 5

✓ **+ 5 pts Correct**

+ 2 pts Euler cycle criterion

+ 1 pts Euler cycle criterion (missing connected or mistake)

+ 2 pts Reduction to even degree graph

+ 1 pts Reduction to even degree graph (with mistake)

+ 1 pts Correct explanation of how to get Euler path from Euler cycle

+ 0 pts Incorrect

3.2 application of euler paths 6 / 5

✓ **- 0 pts Correct**

+ 2 pts Click here to replace this description.

✓ + 1 pts **Click here to replace this description.**

QUESTION 4

4 pigeon hole 10 / 10

✓ - 0 pts **Correct**

- **- 3 pts** Wrong partition
- **- 3 pts** No Pigeonhole
- **- 2 pts** Minor error
- **- 10 pts** Blank

QUESTION 5

hypercube 10 pts

5.1 recurrence for edges 4 / 4

✓ - 0 pts **Correct**

- **- 4 pts** empty
- **- 1 pts** Sounds like you are saying the n-1 dim hypercube has n-1 vertices
- **- 1 pts** need to explain how the hypercube is built out of smaller ones
- **- 4 pts** empty
- **- 0.5 pts** How do you get the recurrence?
- **- 2 pts** large mistake
- **- 2 pts** you are assuming the desired conclusion
- **- 3 pts** can't just do examples

5.2 formula for edges 5 / 6

- **- 0 pts** Correct
- **- 4 pts** wrong answer, need to use iteration

✓ - 1 pts **need to use induction to show that answer is true**

- **- 3 pts** wrong answer, this is why you need to use induction to show answer is true
- **- 6 pts** empty
- **- 4 pts** incomplete
- **- 2 pts** wrong answer

J Cameron Math 61 Mon, February 25th, 2019

Midterm 2

Name: Catherine Xu

Student ID: 905 104 724

Section:

	Tuesday:	Thursday:	
	1A	1B	TA: Albert Zheng
	1C	1D	TA: Benjamin Spitz
	1E	1F	TA: Eilon Reisin-Tzur

Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. **You may not use calculators,** books, notes, or any other material to help you. Please make sure **your phone is silenced** and stowed where you cannot see it. Remember that you are bound by a conduct code, and that you may not look at anyone's paper or let anyone look at your paper, nor may you work on the exam after it has ended. There is some scratch paper at the back of the exam, if you do work there indicate so on the relevant question. Do not write on the backs of the pages. Please circle or box your final answers.

Please get out your id and be ready to show it when you turn in your exam.

Please do not write below this line.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (10 points) Circle the correct answer (only one answer is correct for each question)

1. If X is a set with 10 elements and Y is a set with 3 elements and $f : X \to Y$:

   $\lceil \frac{10}{3} \rceil = 4$

   ~~(a) f could be one-to-one~~

   (b) it is possible that there are *not* 4 distinct elements of X, x_1, x_2, x_3, x_4 with $f(x_1) = f(x_2) = f(x_3) = f(x_4)$.

   (c) there are at least 5 distinct elements of X, x_1, x_2, x_3, x_4, x_5 with $f(x_1) = f(x_2) = f(x_3) = f(x_4) = f(x_5)$.

   (d) there are at least 4 distinct elements of X, x_1, x_2, x_3, x_4 with $f(x_1) = f(x_2) = f(x_3) = f(x_4)$.

2. The number of relations on a set with n elements that are both symmetric and reflexive is:

   ~~(a) $2^{n^2} + 2^{\frac{n^2+n}{2}} - 2^{\frac{n^2-n}{2}}$~~

   ~~(b) $2^{n^2-n} + 2^{\frac{n^2+n}{2}} - 2^{\frac{n^2-n}{2}}$~~

   ~~(c) 2^{n^2}~~

   (d) none of the above

   2^{n^2-n}

   has to be both at the same time according to TA

3. K_n has an Euler cycle:

   (a) For no n

   (b) For n odd

   (c) For n even

   (d) For every n

Question 1 continues on the next page...

Question 1 continued...

$s_3 = 3$

4. If $s_0 = 2$ and $s_1 = 1$ and for $n \geq 2$ $s_n = s_{n-1} + s_{n-2}$ then $s_n =$

   (a) $\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$

   (b) $\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n$

   (c) $\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$

   (d) none of the above

5. The number of ways of arranging a math book, a computer science book, and an English book on a shelf is: 4 books

   (a) $4 - 3 + 2 - 1$

   (b) $4!$

   (c) $1 + 2 + 3 + 4$

   (d) 4

2. In this question write down your answer, no need for any justification. Leave your answers in a form involving factorials, $P(n, m)$, $\binom{n}{m}$, exponents, etc.

(a) (2 points) What is the coefficient of x^7y^3 in the polynomial $(x+2y)^{10}$?

$C(10,7)(1)^7(2^3) = C(10,7)2^3$

(b) (2 points) What is the number of ways of rearranging the letters of the word "MISSISSIPPI"?

M I S P
4 4 2

$\frac{11!}{4!4!2!}$

(c) (2 points) I have 10 distinct books. Two are by Ernest Hemingway and three are by Virginia Woolf. How many ways are there to arrange these books on a shelf if the ones by Hemingway must be next to each other and the ones by Woolf must not be next to each other?

5 not either

9,
6 ⇒ 7 spaces
9-3=6

$P(7,3)P(6,6)$

(d) (2 points) What is the number of binary strings of length 6 that don't contain 11 as a substring?

10101
010101
way 3

$C(6,0) + C(6,1) + [C(6,2) - 5] + [C(6,3) - 4 - 3 - 2 - 1]$

(e) (2 points) What is the number of solutions to the equation $x_1 + x_2 + x_3 = 20$ where $x_3 \leq 10$ and x_1, x_2, x_3 are nonnegative integers?

↳ must be at least 1

$C(17+2,2) - C(7+2,2)$
$C(19,2) - C(9,2)$

$x_1 + x_2 + x_3 = 17$ $\quad x_3 \leq 9$ $\quad x_1 \geq 0$, etc

$C(17+2,2)$

3. (a) (5 points) Show that if a connected graph has exactly 2 vertices of odd degree, then there is a path in the graph that visits each edge exactly once. (Feel free to use what we know about Euler cycles from lecture)

for the 2 vertices of odd degree (let's denote them v_1, v_2), draw an edge connecting the two

as the graph G' is now connected with all even degrees, there is an Euler cycle

say we follow an Euler cycle from v_1 to v_1 with the new edge we created as the last edge we traverse

taking out the new edge from the path we created, we have a path that visits each edge in the original graph exactly once and ending on v_2

Question 3 continues on the next page...

(b) (5 points) A house has a front door and a back door and no other doors leading outside. Inside each room of the house, there are (exactly) two doors that each go to other rooms of the house [1] (and the only way to enter or leave a room is through a door). Each room can be reached from the front door. Show that it is possible to go in through the front door and leave through the back door while going through each door in the house exactly once. Feel free to use the previous part of this problem.

we can define a graph where the rooms are vertices and the front and back doors are vertices too and the door that leads from room 1 to room 2 is an edge incident on the two rooms each room, with the front door connected to all the rooms by a path

this means all the rooms except the rooms just inside the front and back doors have a ~~degree~~ degree of 2, while the ~~the~~ two rooms just inside the front and back doors have a degree of 3

as this graph is connected and has all even degrees except for the rooms just inside the front and back door and the 2 vertices that represent the front and back doors, we can draw an Euler path from the room just inside the front door to the room just inside the back door; adding the single edge from the front door to the room inside the front door ~~and~~ to the start of the path and adding the single edge from just inside the back door to the back door vertices will allow us to go in through the front and leave through the back while going through each door exactly once

[1] To clarify the rooms just inside the front and the back doors have three doors, one of them is the front/back door, and the other two are the doorways leading to other rooms of the house

4. (10 points) Suppose that six distinct integers are selected from the set $\{1,2,3,4,5,6,7,8,9,10\}$. Prove that at least two of these six integers sum to 11.

10

6 integers $a_1, \ldots, a_6$

if two don't add up to 11

$\{a_1, \ldots a_6\} \cap \{11-a_1, \ldots, 11-a_6\} = \emptyset$

however, by the pigeonhole principle

$\left\lceil \frac{6+6}{10} \right\rceil = 2$ so at least one value

from $\{a_1, \ldots, a_6\}$ is also in $\{11-a_1, \ldots, 11-a_6\}$

$\{(1,10), (2,9), \ldots,$

2^n vertices

5. Recall the n-dimensional hypercube. This is a graph with vertices labeled by binary strings of length n, with an edge between two vertices if they differ in exactly one digit. Let c_n be the number of edges in the n-dimensional hypercube.

   (a) (4 points) Show that c_n satisfies the recurrence $c_n = 2^{n-1} + 2c_{n-1}$

for a n-cube, the number of vertices is 2^n

c_{n-1} is the # of edges in a n-1 cube, we make a copy of that cube and connect the ~~[illegible]~~ corresponding same vertices (11 is connected to 11, then we add 0 to one 11 and 1 to the other)

$c_n = 2(\text{\# edges in a } n-1 \text{ cube}) +$ ~~# edges~~ vertices originally in the n-1 cube

$= 2c_{n-1} + 2^{n-1}$

proving by induction

base case $n=1$ $\quad c_n = 1$

$c_1 = 2^{1-1} + 2c_0 = 1 + 2(0) = 1$ ✓

inductive step, assume $c_n = 2^{n-1} + 2c_{n-1}$

for c_{n+1}, we find the number of edges by adding 2 times the previous # of edges to the number of vertices the last cube had

see top for reasoning

$c_{n+1} = 2c_n + 2^n$

So by induction the equation is true

Question 5 continued...

(b) (6 points) Solve your recurrence from the previous part of this question to find a formula for c_n.

$$c_n = 2^{n-1} + 2c_{n-1}$$

$$= 2^{n-1} + 2(2c_{n-2} + 2^{n-2})$$

$$= 2^{n-1} + 2^{n-1} + 2^2(2c_{n-3} + 2^{n-3})$$

$$= (n-1)(2^{n-1}) + 2^{n-1} c_{n-(n-1)}$$

$$= (n-1)(2^{n-1}) + 2^{n-1} c_1$$

$$c_1 = 1$$

$$c_n = (n-1).2^{n-1} + 2^{n-1} = n2^{n-1}$$

This page has been left intentionally blank. You may use it as scratch paper. It will not be graded unless indicated very clearly here and next to the relevant question.